Parametric Reconstruction of Kinetic PET Data with Plasma Function Estimation

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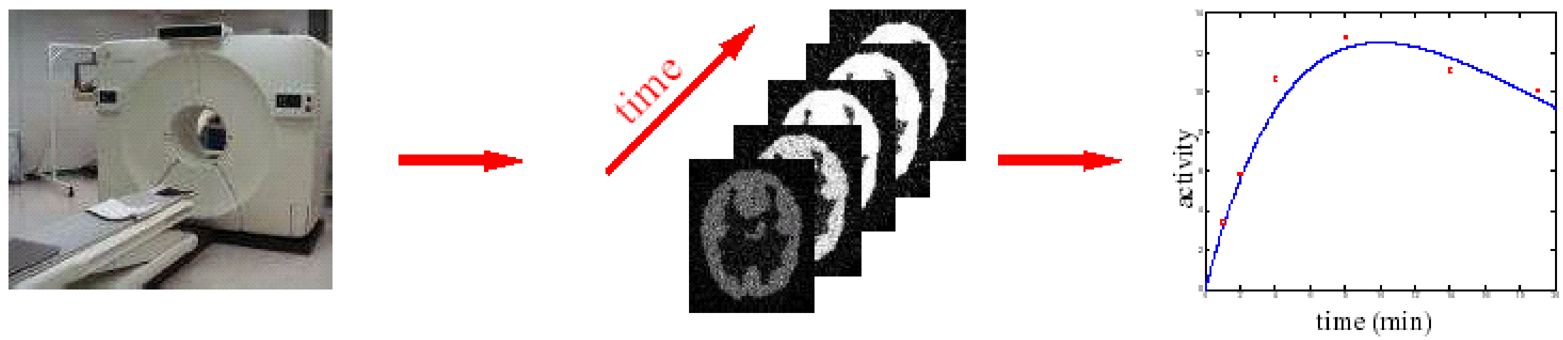
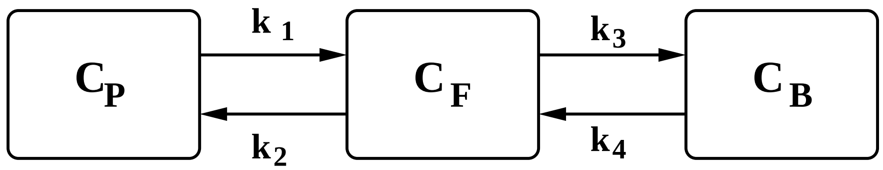
Dynamic PET



- Time response of tracers is required in imaging heart perfusion, brain activation, glucose metabolization, receptor availability
- Time resolution can be achieved by dividing the data into time frames.
- Time response of voxels are governed by ODEs
- Parameters of these ODEs are physiologically relevant

2-tissue Compartment Model Equations



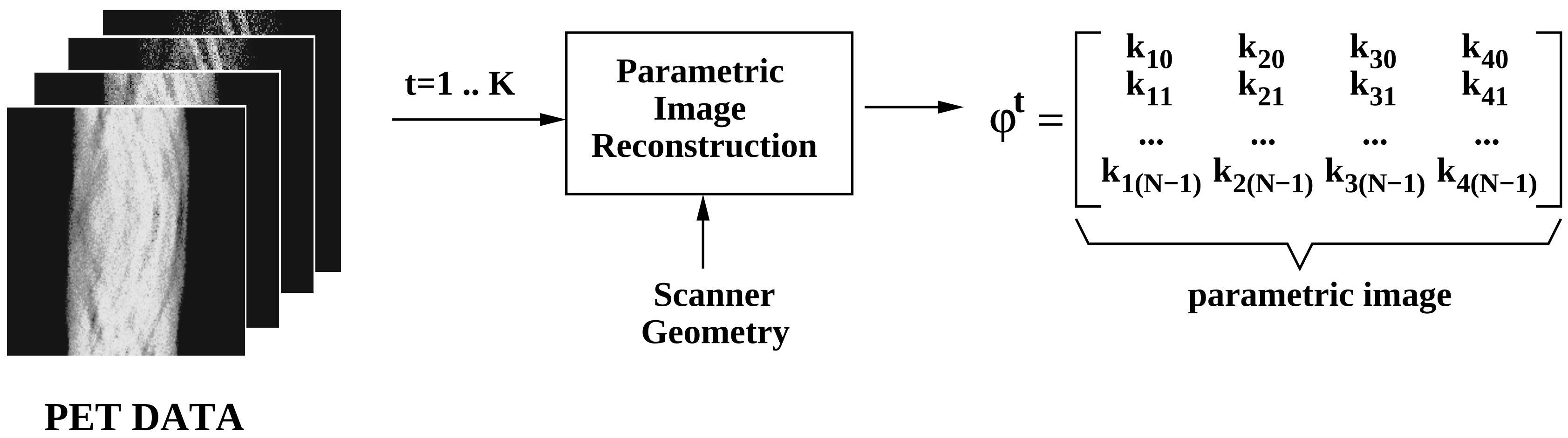
- C_P is assumed to be
  - known
  - same for all voxels
- Kinetic parameters vary for each voxel location
- Time variation of molar tracer concentrations at voxel s

$$\frac{dC_F(t)}{dt} = k_{1s}C_P(t) - (k_{2s} + k_{3s})C_F(t) + k_{4s}C_B(t)$$
$$\frac{dC_B(t)}{dt} = k_{3s}C_F(t) - k_{4s}C_B(t)$$

- PET signal at voxel s,

$$f(\varphi_s, t) = (1 - V_B)(C_F(t) + C_B(t))S_A e^{-\lambda t} + V_B C_{WB}(t)$$

Our Approach: Direct Parametric Image Reconstruction



- Advantages:
  - Directly reconstructs parameters for each voxel from sinogram data
  - Regularization on physiologically relevant parameters
  - Dimensionality reduction
  - Improves SNR

Scanner Model

- Y sinogram matrix; Y_{mk} independent Poisson distributed
- A forward model; A_{ij} detection probability of an event from voxel j by detector pair i
- $\varphi \rightarrow$ parametric image
- $F(\varphi, t_k) \rightarrow$ time response of all voxels at k^{th} time frame
- Log likelihood

$$
\begin{aligned}
LL(Y|\varphi) \quad &= \quad \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} Y_{mk} \log(A_{m*}F(\varphi, t_k) + \mu) \\
&\quad -(A_{m*}F(\varphi, t_k) + \mu) - \log(Y_{mk}!)
\end{aligned}
$$

MAP Estimate of Parametric Image

$$
\begin{aligned}
C(Y|\varphi) &= -LL(Y|\varphi) + S(\varphi) \\
\hat{\varphi} &= \arg\min_{\varphi} C(Y|\varphi)
\end{aligned}
$$

- How do we efficiently compute $\hat{\varphi}$?
- How to choose $S(\varphi)$?

PICD - Parametric Iterative Coordinate Descent

- Efficient implementation of ICD for reconstruction with kinetic models
- We re-parametrize using $\varphi_s = [a_s,\ b_s,\ c_s,\ d_s]^t$

$$f(\varphi_s, t) = (1 - V_B)\left[(a_s e^{-c_s t} + b_s e^{-d_s t})u(t) * C_P(t)\right] S_A e^{-\lambda t} + V_B C_{WB}(t)$$

- Sequentially update parameter φ_s vector at each voxel

$$\varphi_s \leftarrow C(Y|\varphi_s)$$

- $C(Y|\varphi)$ decreases with each PICD iteration

Stabilizing Function, $S(\varphi)$

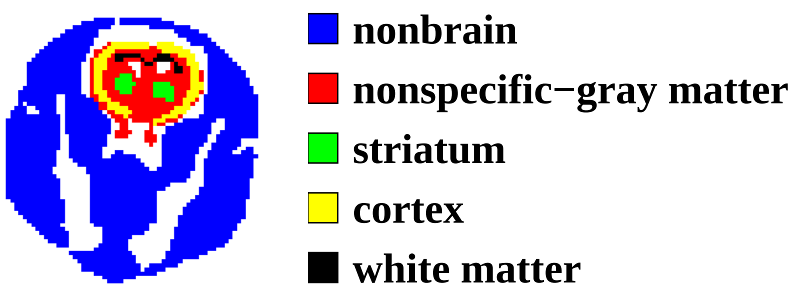
- Model the distribution of parametric image with Markov Random Field (GMRF)
- Negative logarithm of distribution is used as stabilizing function

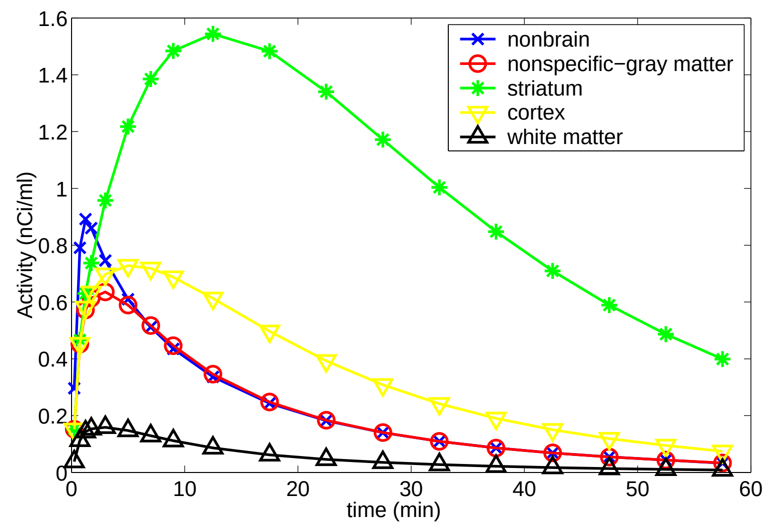
$$S(\varphi) = \sum_{\{s,r\}\in\mathcal{N}} g_{s-r}\|T(\varphi_s) - T(\varphi_r)\|_W^2 \ .$$

- $T(\cdot)$, let you regularize physiologically relevant parameters.

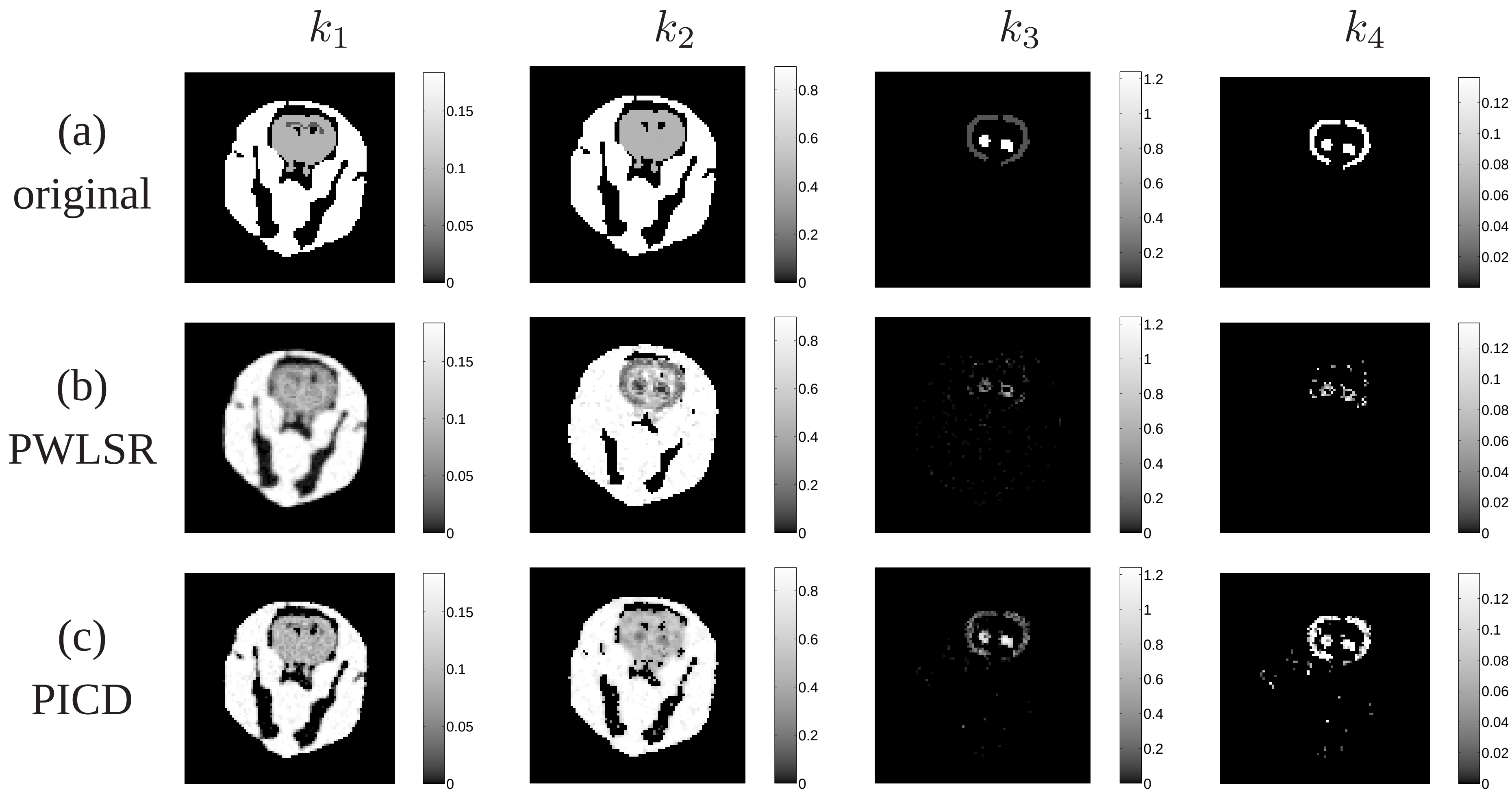
Simulations - Phantom

- Rat phantom with seven separate regions is used to assess the estimation methods
- Regions are obtained by segmenting MRI scans of a rat
- Kinetic parameters of regions are obtained from literature
- Total scan time is 60 min.
- Poisson noise model with 10 M counts

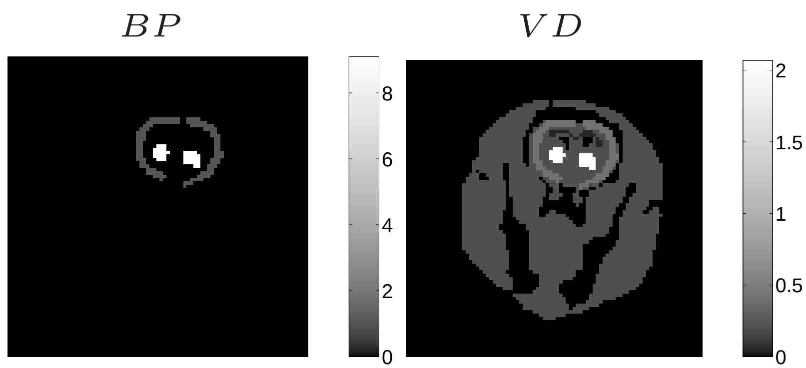




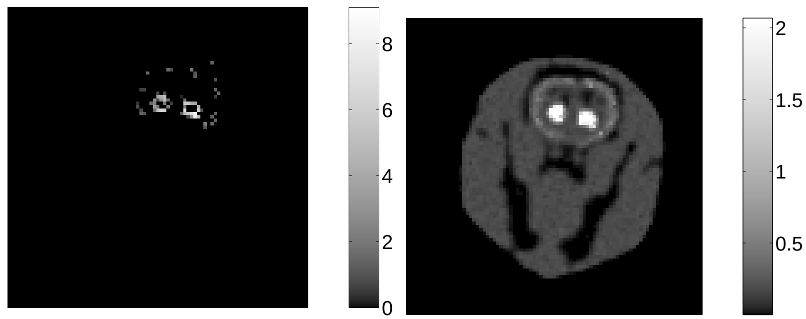
Kinetic Parameter Estimates



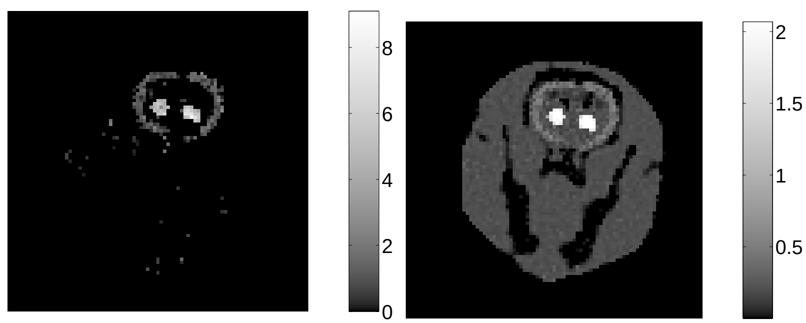
Estimates of Physiologically Important Parameters



(a) Original

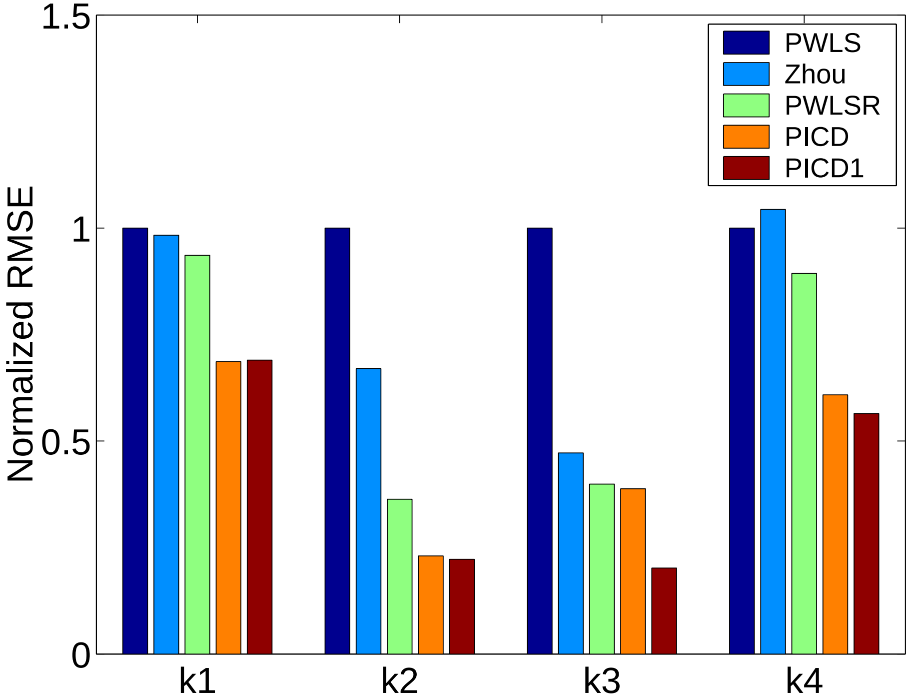


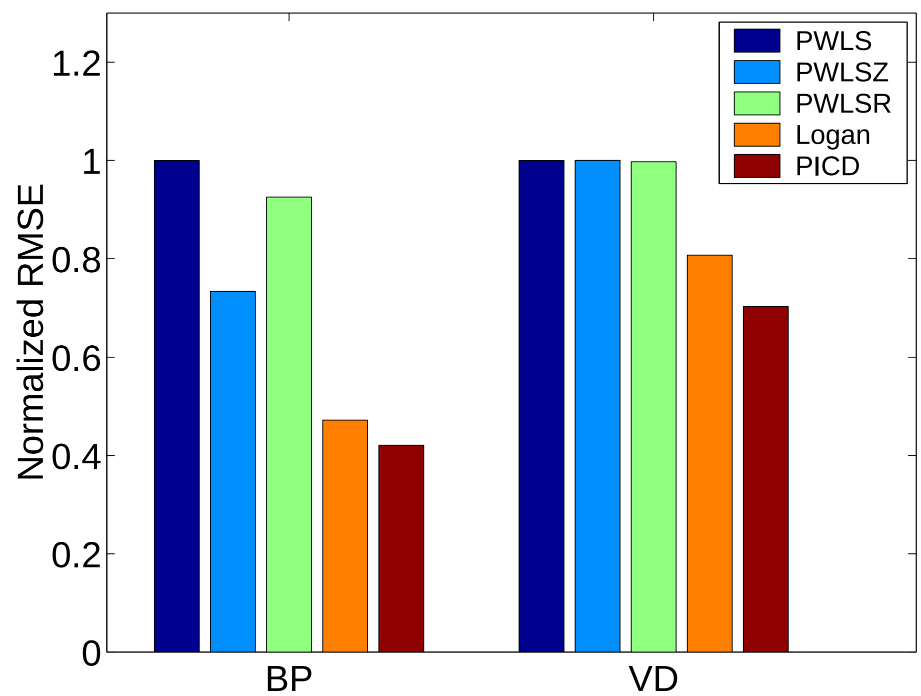
(b) PWLSR



(c) PICD

Normalized RMSE of Estimated Parameters

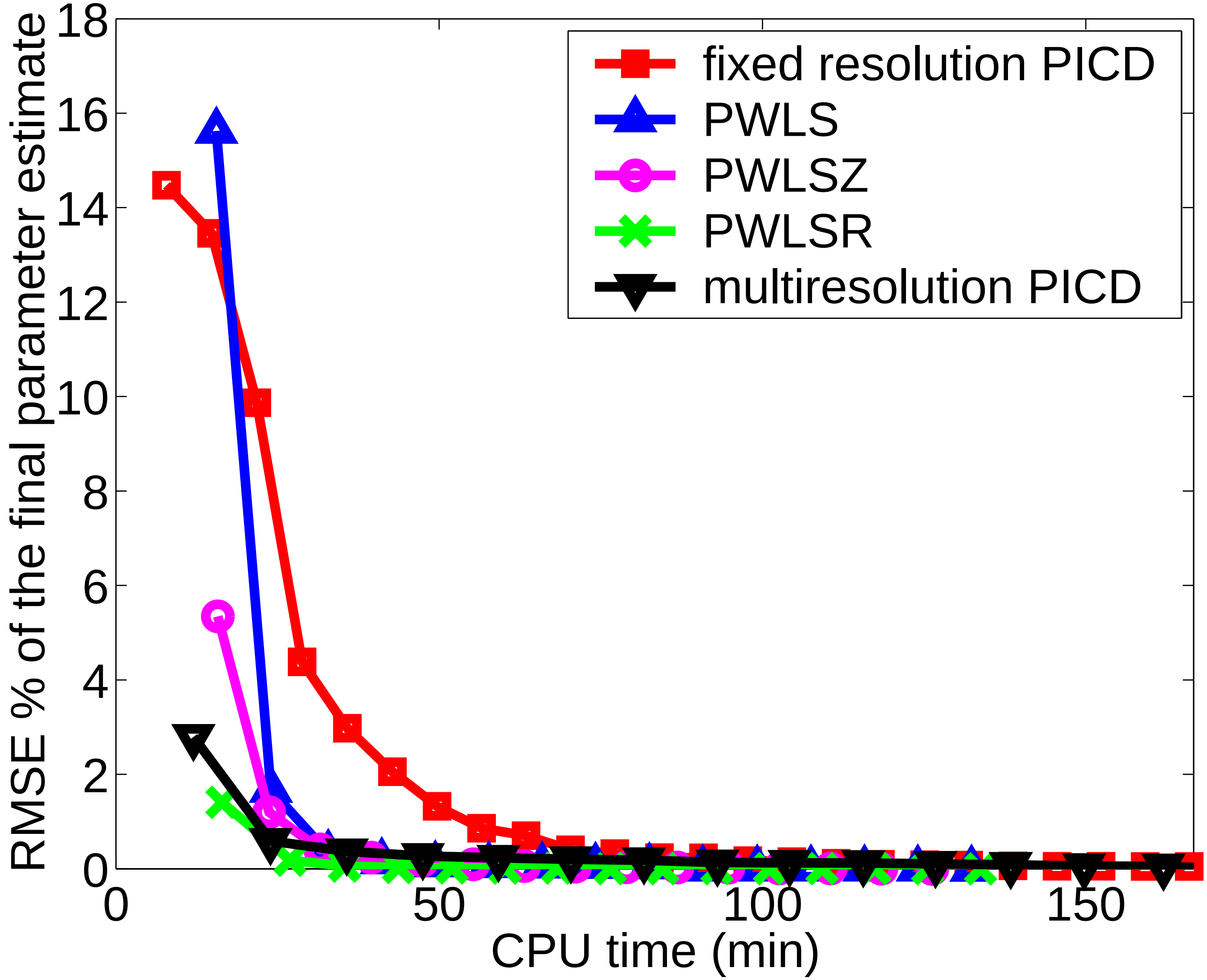




RMSE for

- $k_1, VD \longrightarrow$ over all image
- $k_2, k_3, BP \longrightarrow$ over the support of k_1
- $k_4 \longrightarrow$ over the support of k_3

Convergence



Plasma Input Function Estimation

- Can we estimate plasma model parameters, ϕ, simultaneously with kinetic parameters ?
- Plasma Model with 5 parameters, $\phi = [A_1, A_2, \lambda_1, \lambda_2, \tau]^t$ (Feng *et al.*)

$$C_P(\phi, t) = \left\{ (A_1(t-\tau) - A_2)e^{-\lambda_1(t-\tau)} + A_2 e^{-\lambda_2(t-\tau)} \right\} u(t-\tau)$$

- New cost function

$$C(Y|\varphi, \phi) = -LL(Y|\varphi, \phi) + S(\varphi)$$

- φ, and ϕ can then be estimated

$$\{\hat{\varphi}, \hat{\phi}\} \leftarrow \arg \min_{\{\varphi, \phi\}} C(Y|\varphi, \phi)$$

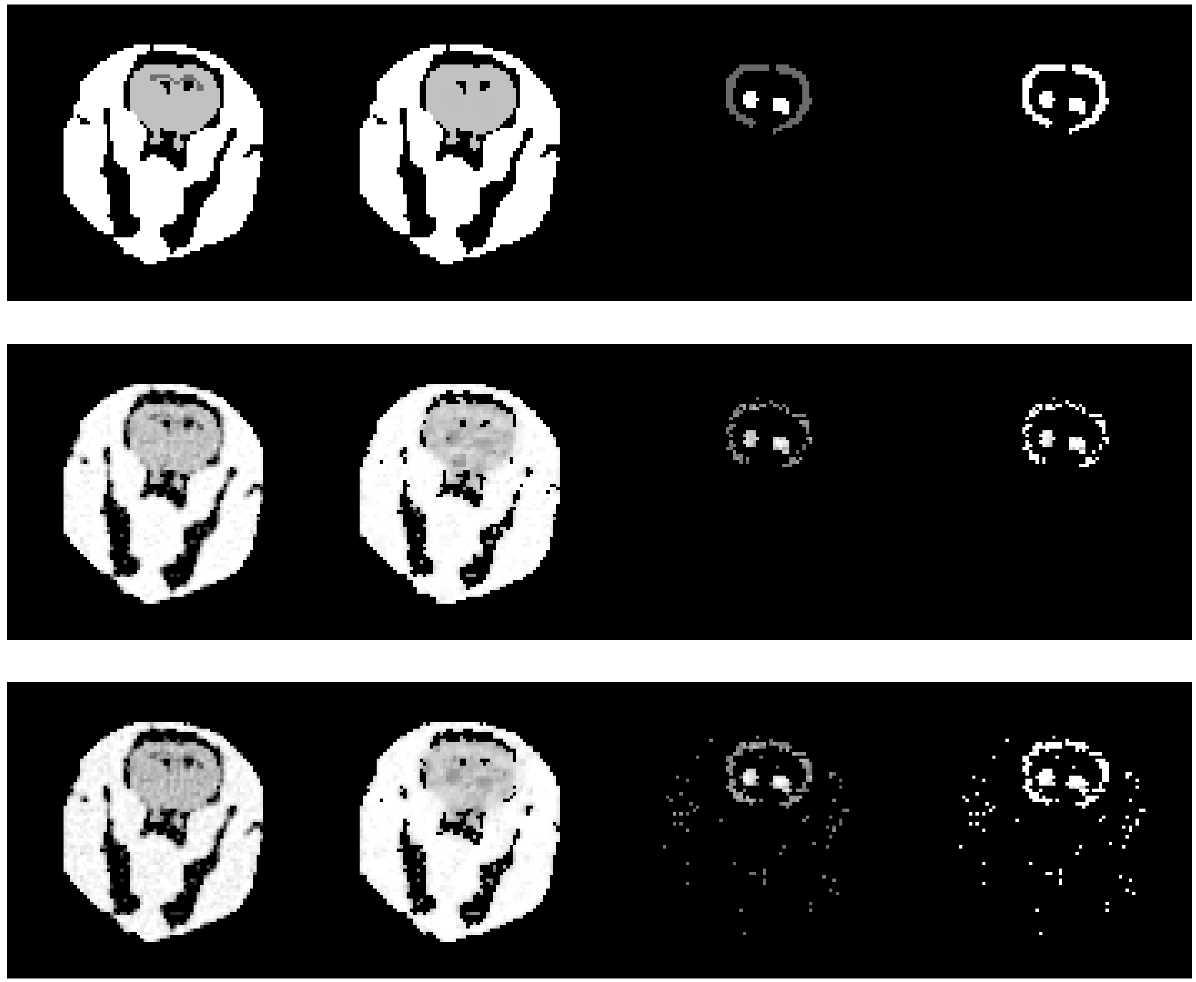
Estimated k_1, k_2, k_3 and k_4

k_1 k_2 k_3 k_4

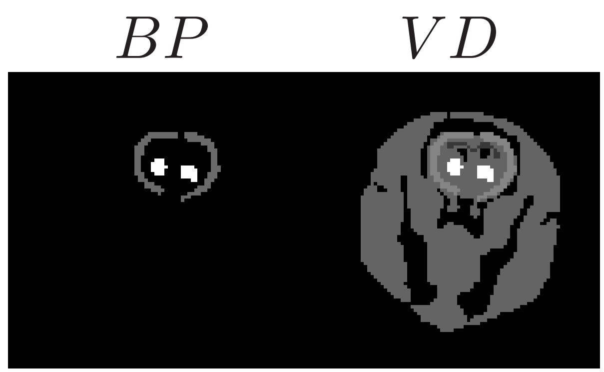
(a) originals

(b) with measured input function

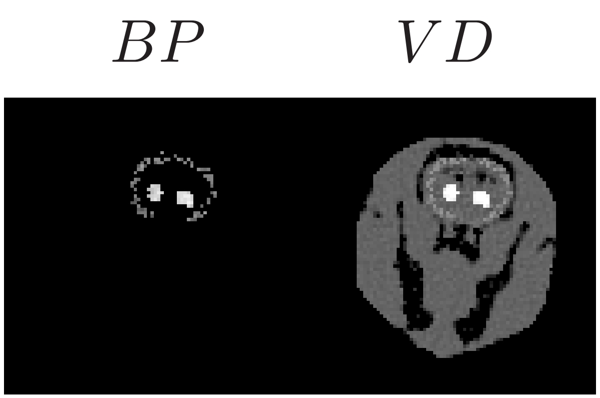
(c) with estimated input function



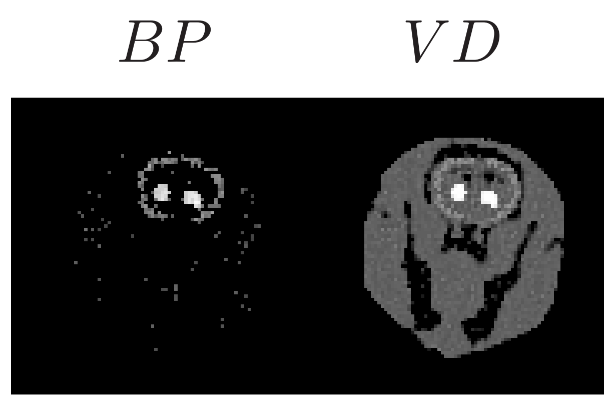
Estimated BP and VD



(a) original

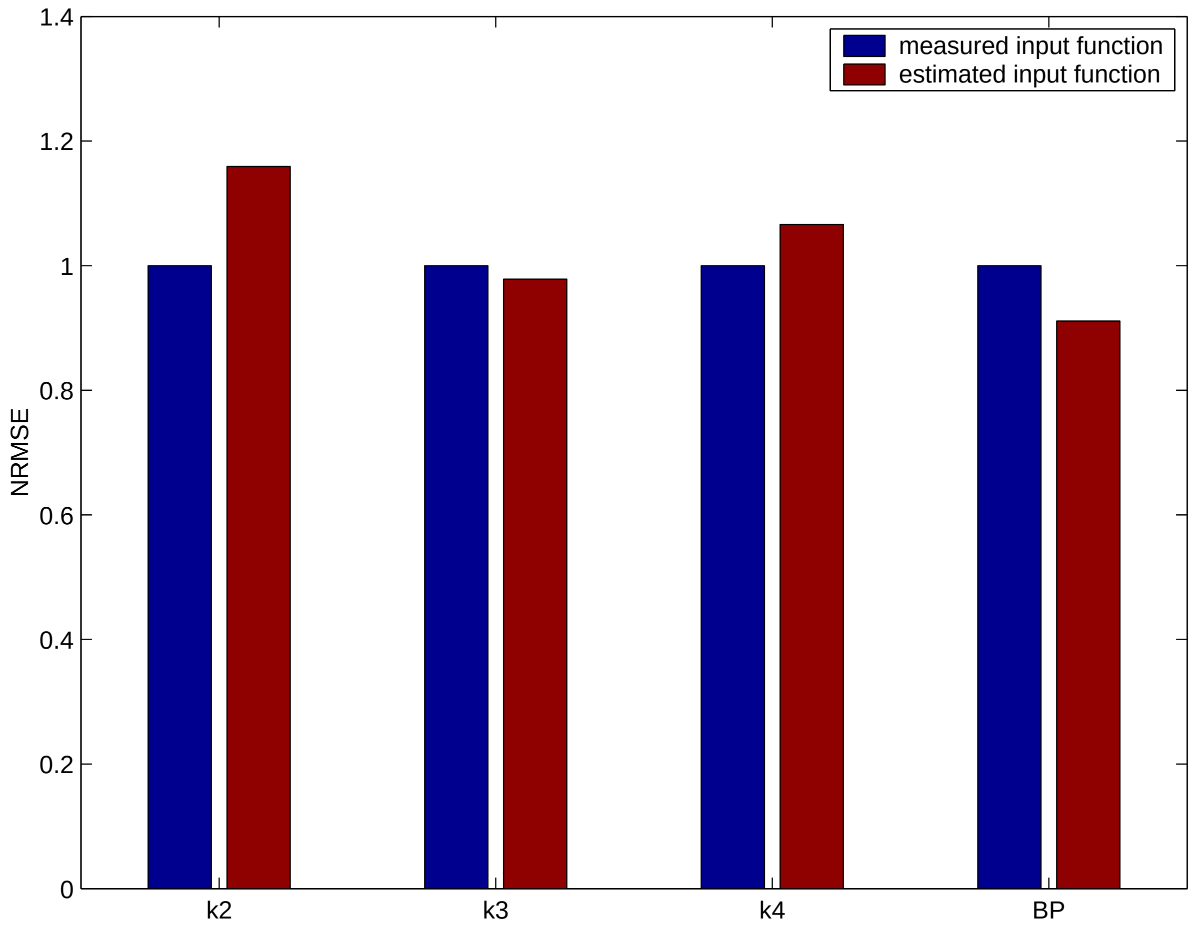


(b) with measured input function



(c) with estimated input function

Normalized RMSE of estimated k_1, k_2, k_3 and k_4



Conclusions

- Propose direct reconstruction of parametric image
- Advantages
  - Dimensionality reduction
  - Higher SNR
  - Dense parameter estimates
- Demonstrated improved quality on realistic simulation data
- Model-based plasma input function estimated simulatenously with kinetic parameters

Thank You !