Direct Reconstruction of Kinetic Parameter Images from Dynamic PET Data

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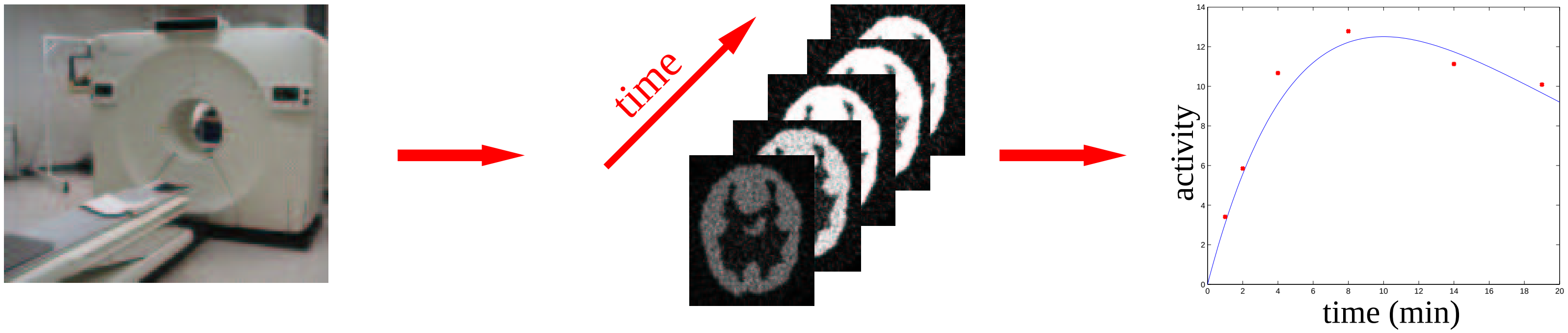
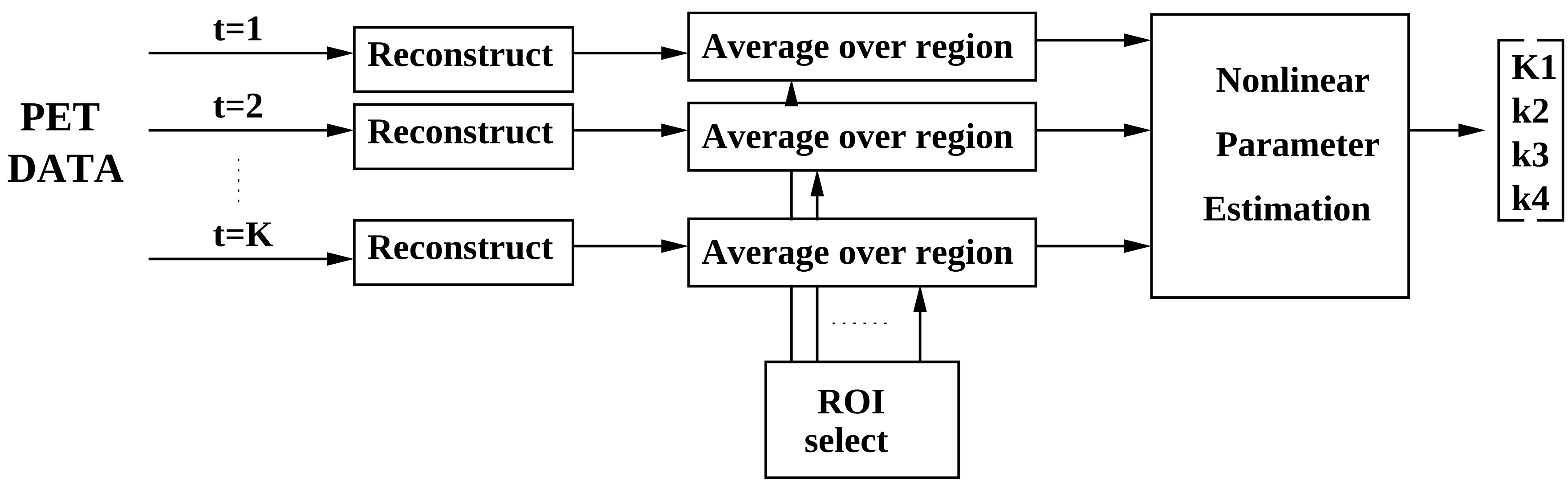
Dynamic PET



- PET accumulates/averages the emissions of voxels.
- Time resolution can be achieved by dividing the data into time frames.
  - Heart perfusion
  - Brain activation
  - Glucose utilization rate
  - Receptor-ligand
- Time response of voxels are governed by ODEs
- Parameters of these ODEs are clinically important

Current Method for Estimation of Compartment Model Parameters

- SNR is low
- Some parameters are nearly unidentifiable
- Current techniques reconstruct time sequences of images and perform parameter estimation on large regions.



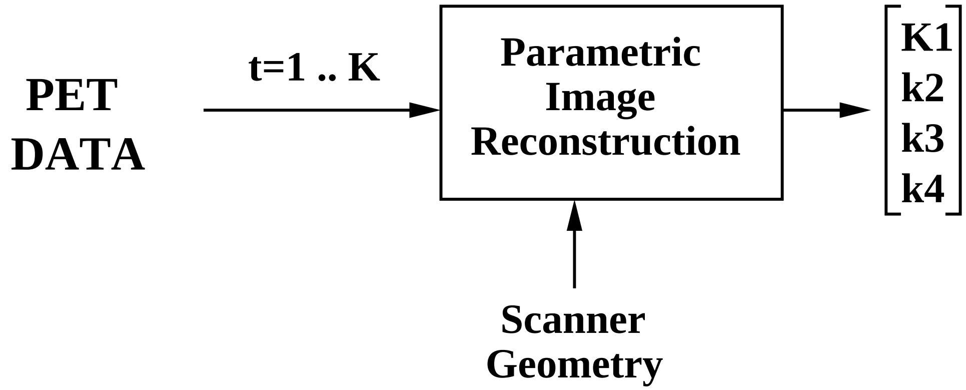
Limitation of Current Approach

- Requires high SNR
- Depends on accurate ROI
- Does not yield dense parametric estimate
- “Partial Volume” effect is a problem
- Requires reconstruction of many low SNR images

Extensions to Dense Parameter Estimation Methods

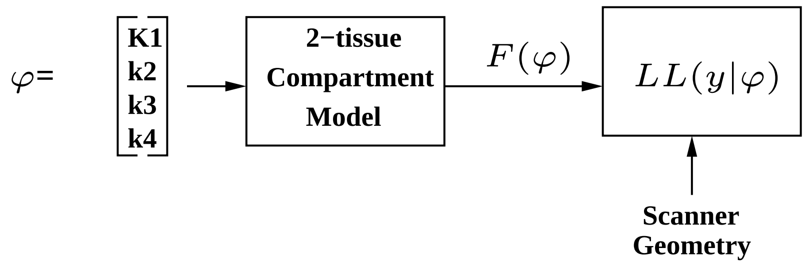
- Pixelwise Weighted Least Squares (PWLS):
  - Each voxel parameter is estimated independently
  - no a priori information
- Pixelwise Weighted Least Squares with regularization (PWLSR):
  - Same as PWLS but with spatial regularization

Our Approach: Parametric Image Reconstruction



- Advantages:
  - Directly reconstructs parameters from sinogram data
  - Improves SNR
  - Dimensionality reduction
  - Produces a single full image of parameter vector
  - Point spread function and system geometry can account for “Partial Volume” effects

Parametric Reconstruction Model



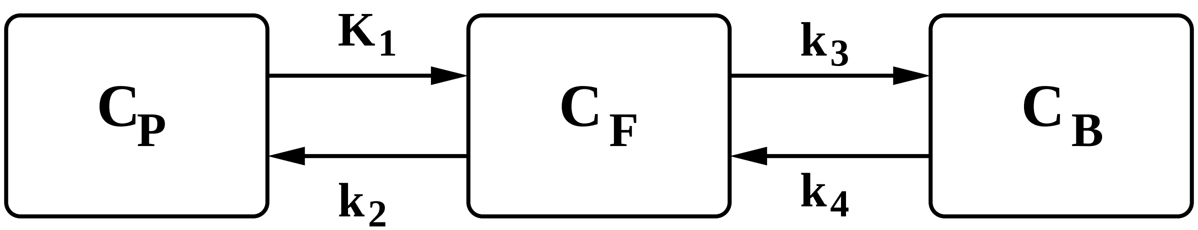
- φ_s parameter vector of voxel s
- $f(\varphi_s) = \begin{bmatrix} f(t_1, \varphi_s) \\ \vdots \\ f(t_K, \varphi_s) \end{bmatrix}$ time response at voxel s
- $F(\varphi) = [f(\varphi_1),\ f(\varphi_2), \ldots, f(\varphi_N)]$ time response of all voxels
- y is the sinogram data
- Log likelihood has the form $LL(y|F(\varphi))$

Compartmental Models

- Models needed to quantify processes
- Parameters of the model correspond to clinically important information
- Compartmental models,
  - use compartments for physical spaces and states of tracer
  - use rate of tracer exchange between compartments as its parameters
  - can be described by first order ODEs
- Complex processes can be modeled by adding more compartments into the model

2-tissue Compartment Model

- Used in;
  - FDG studies
  - Receptor studies



- C_P, plasma compartment: Tracer concentration inside the arterial blood vessels
- C_F, free compartment: Tracer concentration in the tissue that is not metabolized or bounded
- C_B, bound compartment: Tracer concentration in the tissue that is metabolized or bounded

2-tissue Compartment Model Equations

- C_P is measured by sampling blood from the patient during the scan
- Tracer concentration at other compartments

$$\frac{dC_F(t)}{dt} = K_1 C_P(t) - (k_2 + k_3) C_F(t) + k_4 C_B(t) \tag{1}$$

$$\frac{dC_B(t)}{dt} = k_3 C_F(t) - k_4 C_B(t) \tag{2}$$

- PET signal,

$$C_T(t) = C_F(t) + C_B(t) \tag{3}$$

$$f(K_1, k_2, k_3, k_4) = \left[(1 - V_B) C_T(t) + V_B C_P(t)\right] S_A e^{-\lambda t} \tag{4}$$

2-tissue Compartment Model: Important Parameters

- For receptor-ligand imaging *binding potential (BP)* and *volume distribution (VD)* are clinically important parameters.

$$\mathrm{BP} = \frac{k_3}{k_4} \tag{5}$$

$$\mathrm{VD} = \frac{K_1}{k_2}\left(1 + \frac{k_3}{k_4}\right) \tag{6}$$

MAP Estimate of Parametric Image

$$\begin{aligned} C(y|\varphi) &= LL(y|\varphi) + S(\varphi) & (7) \\ \hat{\varphi} &= \arg\max_{\varphi} C(y|\varphi) & (8) \end{aligned}$$

- How do we efficiently compute this

PICD - Parametric Iterative Coordinate Descent

- Efficient implementation of ICD for reconstruction with kinetic models
- Sequentially update parameter φ_s vector at each voxel
- $LL(y|\varphi) + S(\varphi)$ will increase with each PICD iteration
- Efficient when $F(\varphi)$ is a nonlinear function
- Works with MRF prior

PICD - Update Strategy

- For each voxel update, make approximation

$$LL(y|\varphi_s) - LL(y|\tilde{\varphi}_s) \approx \sum_k (\theta_{1k}\Delta f_{sk} + \frac{1}{2}\theta_{2k}\Delta f_{sk}^2) \tag{9}$$

  where $\Delta f_{sk} = f(t_k, \varphi_s) - f(t_k, \tilde{\varphi}_s)$

- θ_{1k} and θ_{2k} can be recursively updated using same algorithm as in conventional ICD [Bouman and Sauer 96]

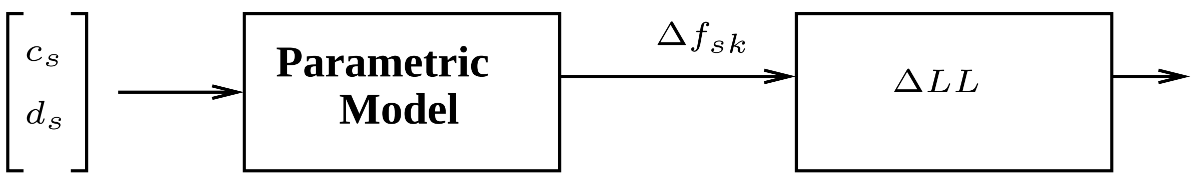
- We re-parametrize using $\varphi_s = [a_s,\ b_s,\ ,c_s,\ d_s]$

- Then the time response is

$$f(t_k, \varphi_s) = \left[(1 - V_B)[(ae^{-ct} + be^{-dt}) \otimes C_P(t_k)] + V_B C_P(t_k)\right] S_A e^{-\lambda t} \tag{10}$$

PICD - Pixel Vector Update

- Estimation of a_s and b_s parameters
  - linear parameters
  - closed form update for fixed values of c_s and d_s
  - dependence on a_s and b_s is removed
- Estimation of c_s and d_s parameters
  - nonlinear parameters
  - $\Delta f_{sk}(c_s, d_s)$



$$
\begin{aligned}
c_n &\leftarrow \arg\max_{c_s \geq d_s} \Delta LL(y|c_s,\ d_s) + S(\varphi) &\quad (11)\\
d_n &\leftarrow \arg\max_{d_s \geq 0,\ d_s \leq c_s} \Delta LL(y|c_s,\ d_s) + S(\varphi) &\quad (12)
\end{aligned}
$$

Multiresolution Reconstruction

- Multiresolution reconstruction
  - Coarsest scale initialized to constant value
  - Coarse scale solutions are used to initialize fine scale solutions
  - Used 3 scales (32×32, 64×64 and 128×128)

Simulations - Phantom

- Rat phantom with seven separate regions is used to assess the estimation methods

Region	k_1	k_2	k_3	k_4	a	b	c	d
Background	0	0	0	0	0	0	0	0
CSF	0	0	0	0	0	0	0	0
Nonbrain (NB)	.1836	.8968	0	0	.1836	0	.8968	0
Whole brain (WB)	.0918	.4484	0	0	.0918	0	.4484	0
Straitum (STR)	.0918	.4484	1.2408	.1363	.02164	.07016	1.7914	.0312
Cortex (COR)	.0918	.4484	.141	.1363	.0607	.0311	.628	.09725
White matter (WM)	.02295	.4484	0	0	.02295	0	.4484	0

- Regions are obtained by segmenting MRI scans of a rat
- Total scan time is 60 min. , divided into 18 time frames: 4×0.5 min, 4×2 min and 10×5min

Simulations - Assumptions

- Raclopride with ^{11}C is used as tracer.
- The blood function, $C_P(t)$ was generated as described in [Wong *et. al. 01*]
- Activity scaled to are scaled 10M counts
- 180 projection angles each with 200 projection and 0.875 mm spacing
- Used 4 mm. wide triangular PSF
- Poisson noise model with accidental coincidences
- Comparison methods use FBP

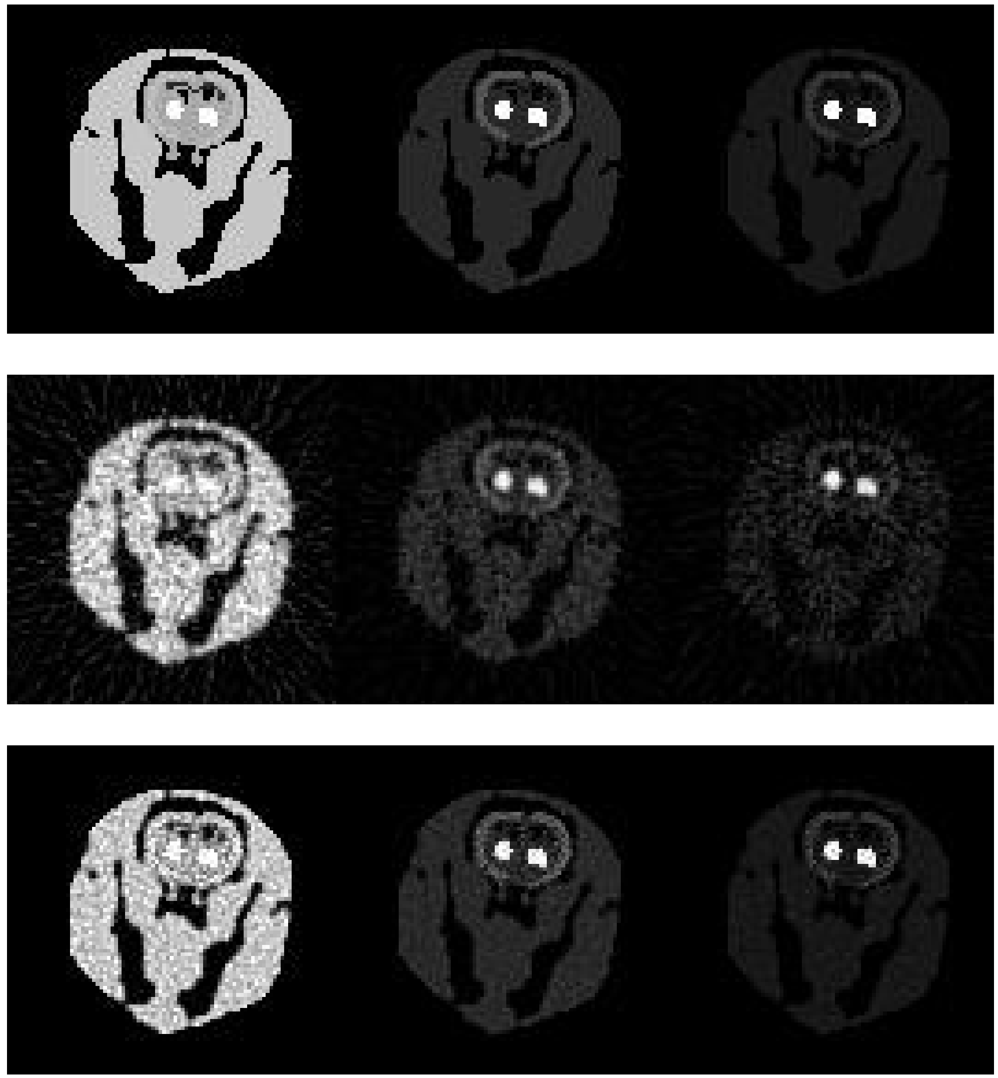
Reconstructed Emission Images

Frame 5 Frame 10 Frame 15

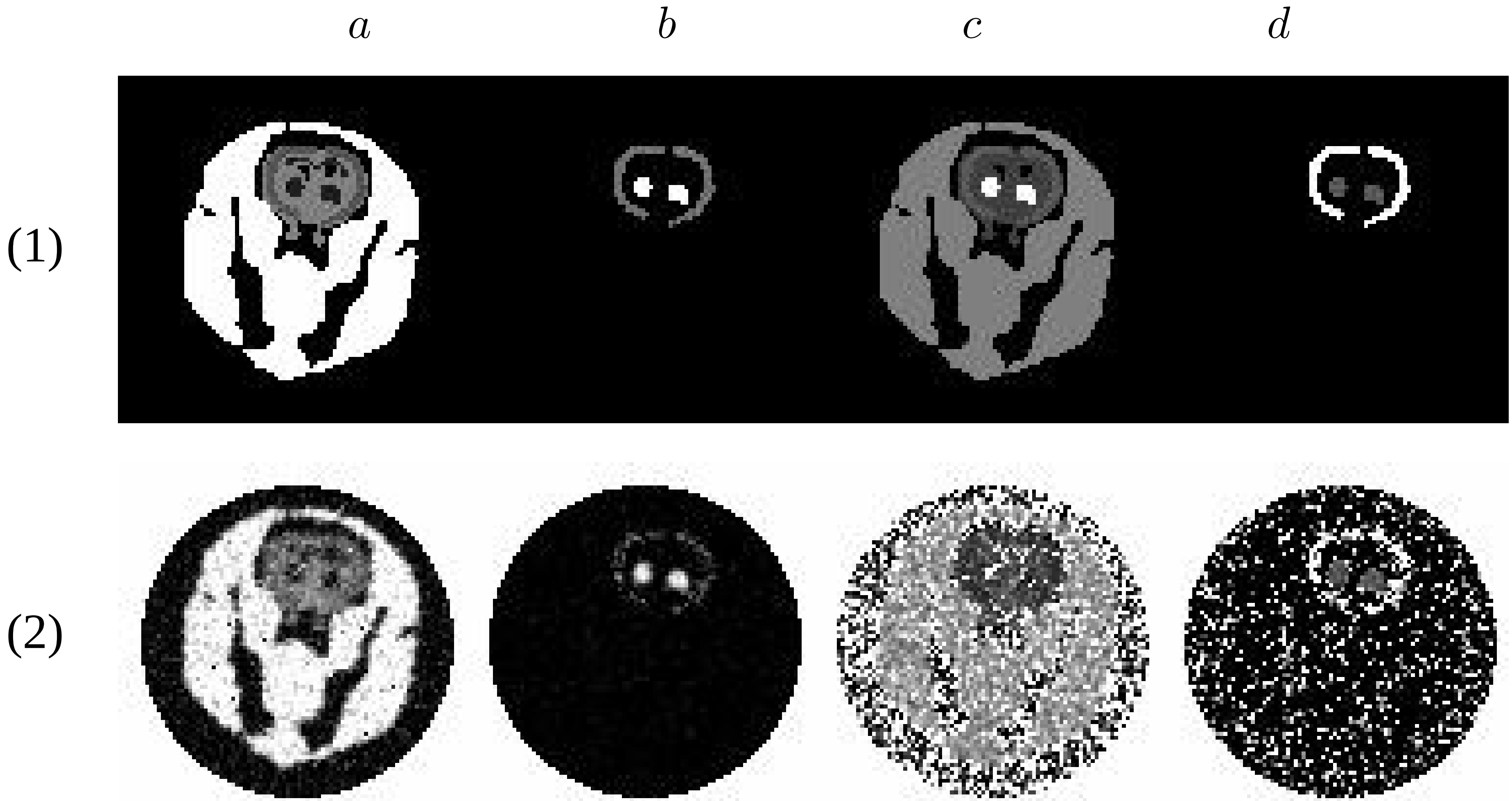
Original phantom

FBP reconstruction

Parametric reconstruction



Parametric Images of a, b, c and d

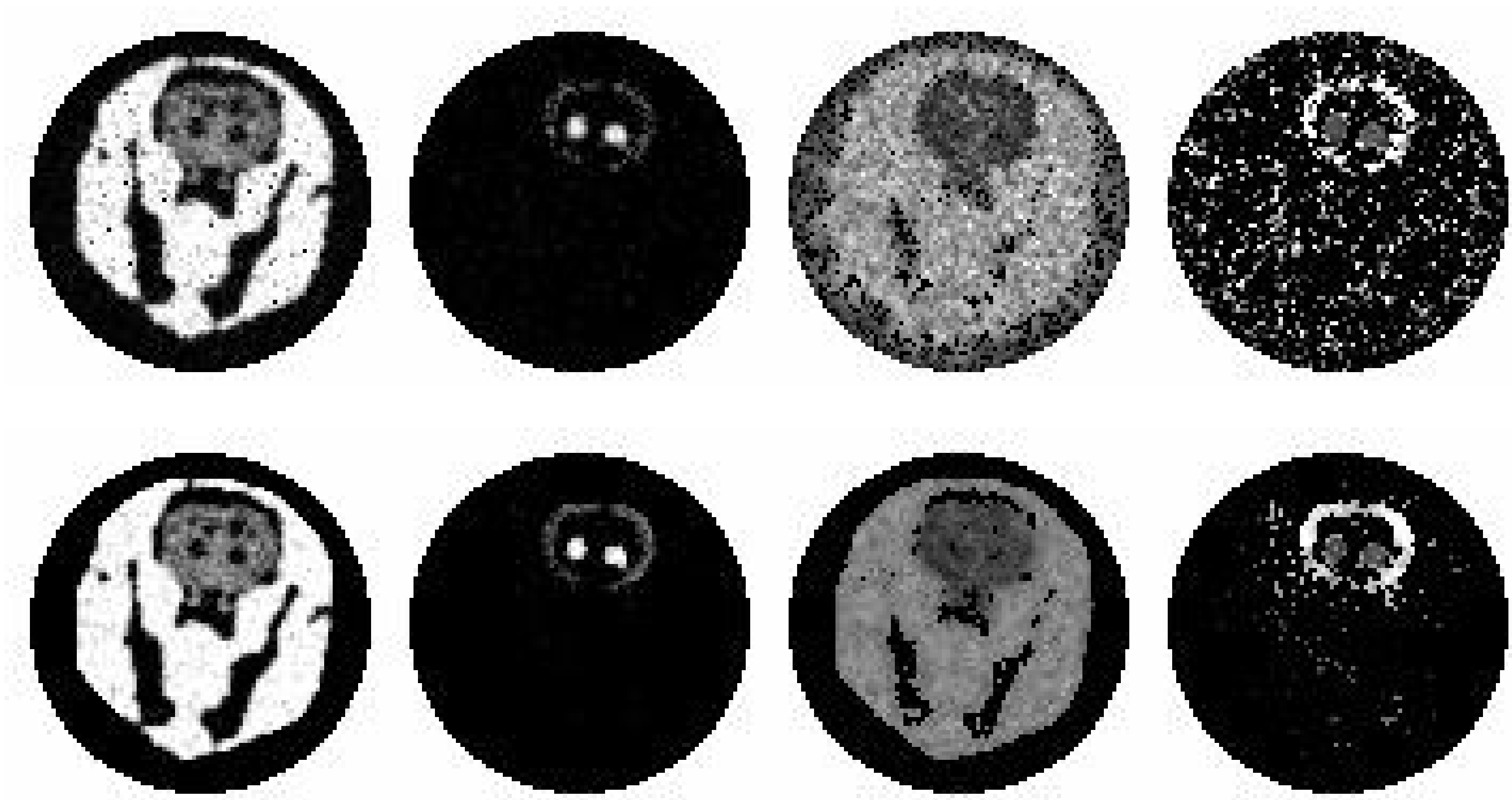


(1) Original Phantom (2) Pixelwise Weighted Least Squares

a b c d

(3)

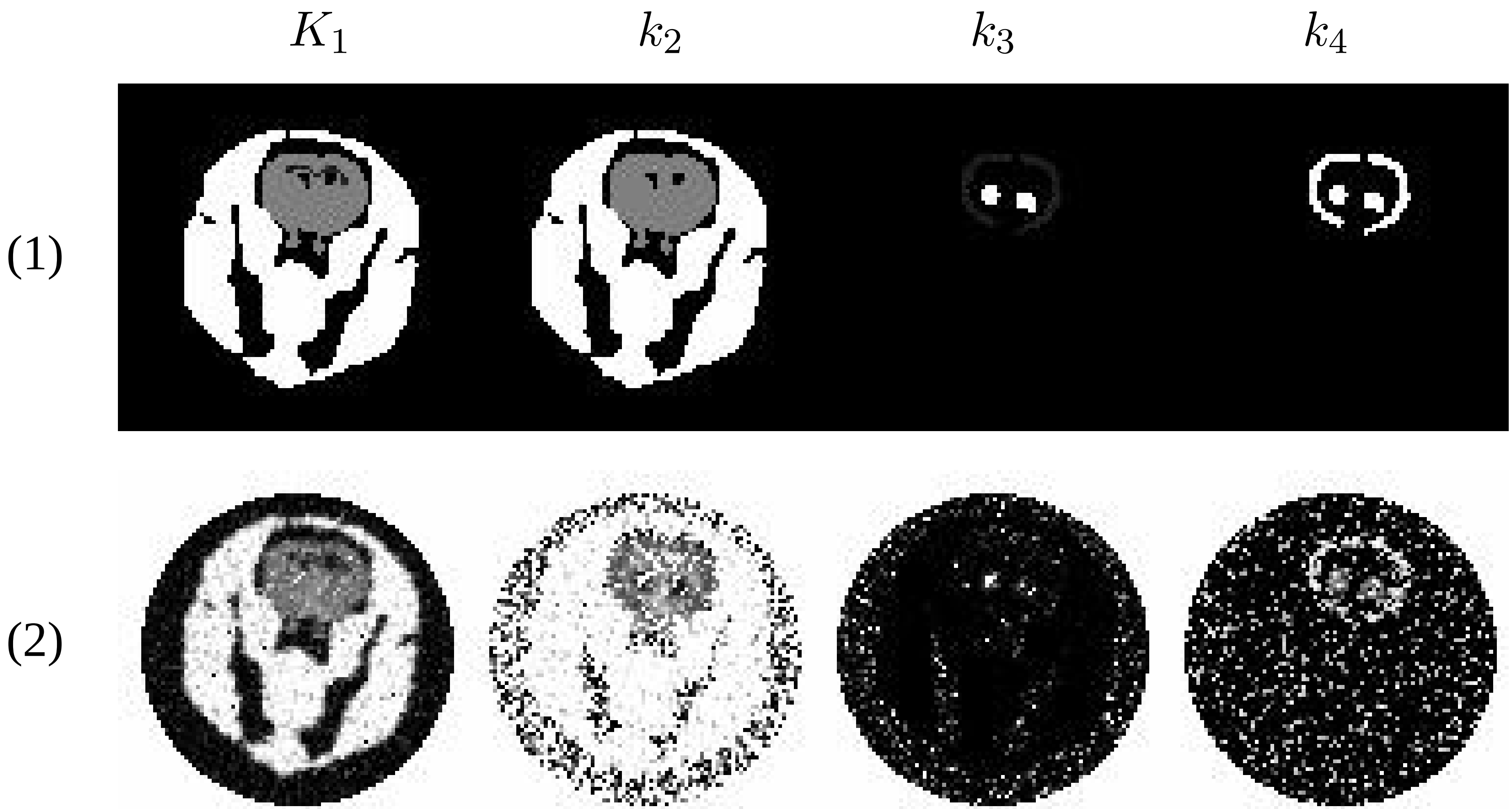
(4)



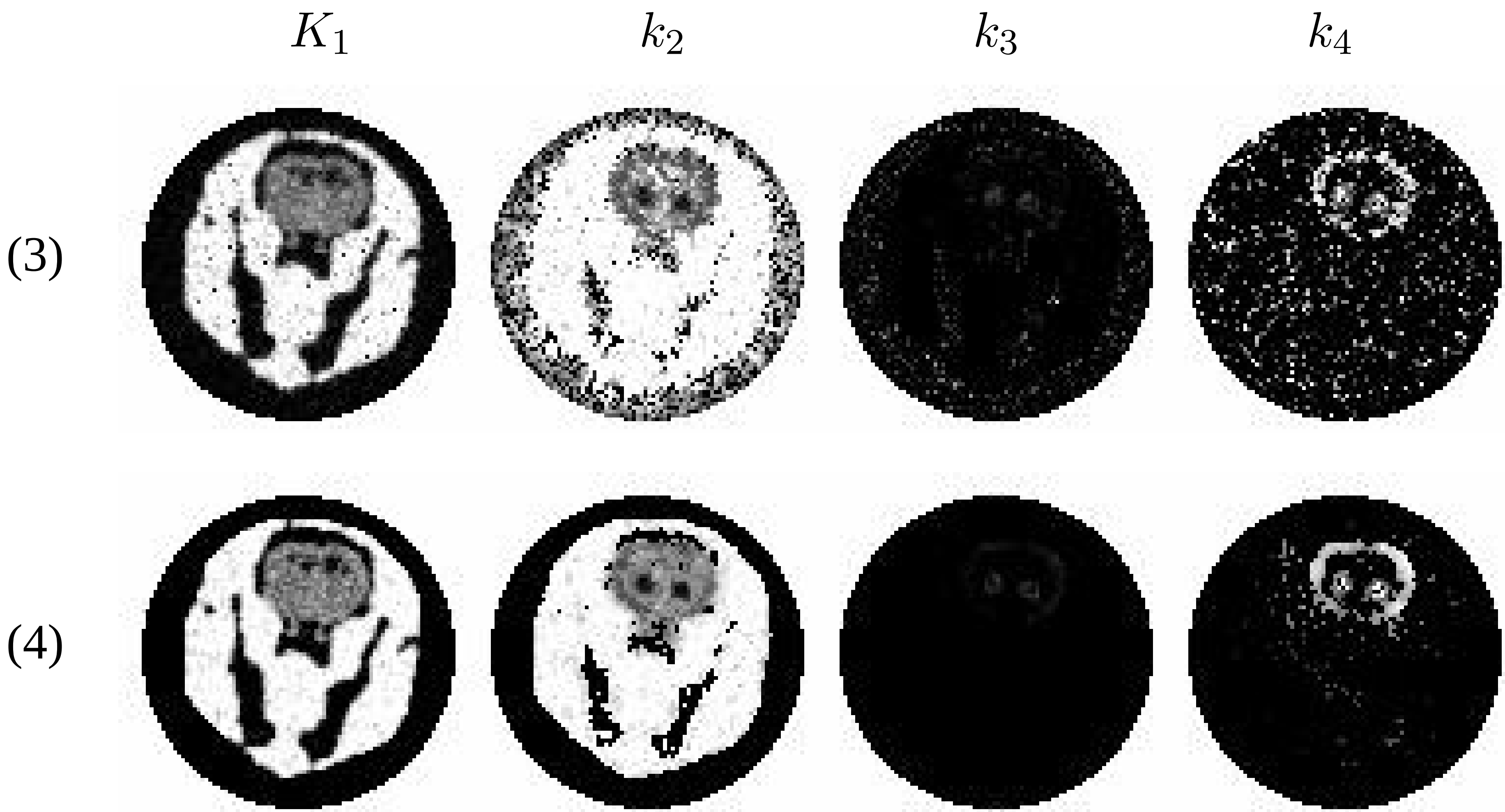
(3) Pixelwise Weighted Least Squares with Regularization

(4) Parametric Image Reconstruction

Parametric Images of K_1, k_2, k_3 and k_4



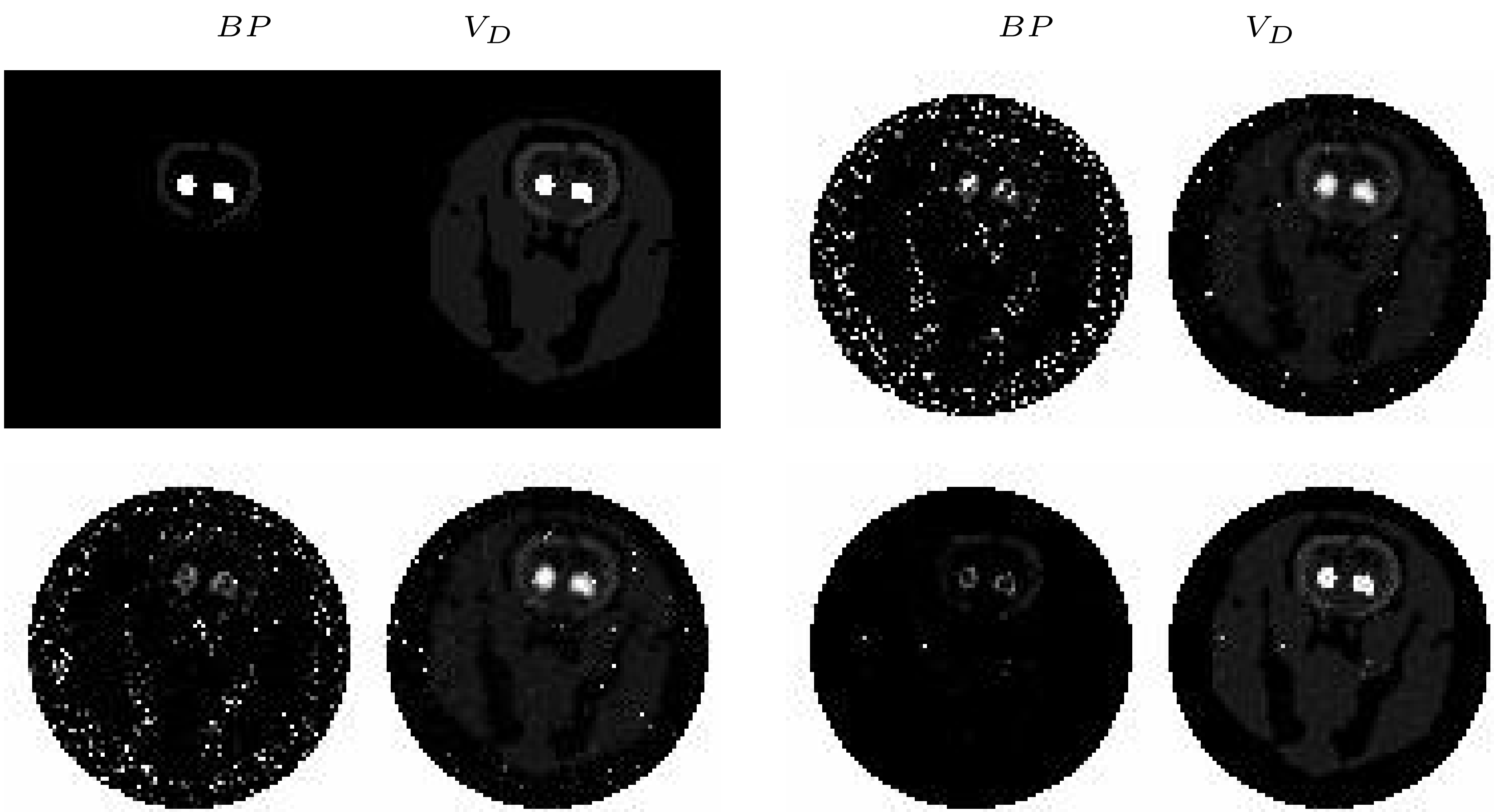
(1) Original Phantom (2) Pixelwise Weighted Least Squares



(3) Pixelwise Weighted Least Squares with Regularization

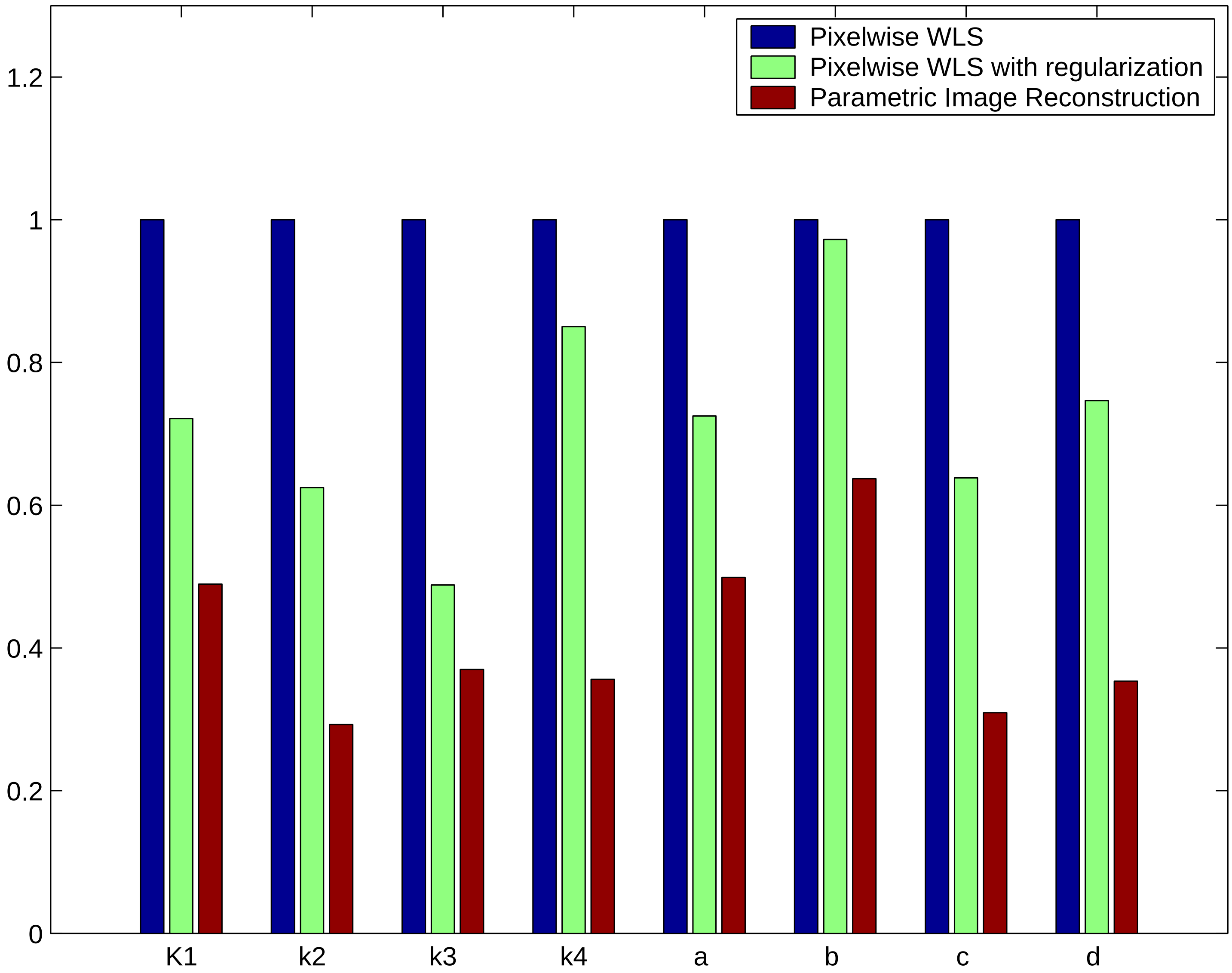
(4) Parametric Image Reconstruction

Parametric Images of BP and V_D



(1) Original Phantom	(2) Pixelwise Weighted Least Squares
(3) Pixelwise Weighted Least Squares with Regularization	(4) Parametric Image Reconstruction

Normalized RMSE of the Parametric Images



Conclusions

- Propose direct reconstruction of parametric image
- Advantages
  - Higher SNR
  - Dense parameter estimates
  - Reduced "Partial Volume" effect
- Demonstrated improved quality on realistic simulation data

References

[1] C. A. Bouman and K. Sauer, "A unified approach to statistical tomography using coordinate descent optimization," *IEEE Trans. on Image Processing*, vol. 5, no. 3, pp. 480–492, March 1996.

[2] Koon-Pong Wong, Dagan Feng, Steven R. Meikle, and Michael J. Fulham, "Simultaneous estimation of physiological parameters and the input function - in *vivo* pet data," *IEEE Transactions on Information Technology in Biomedicine*, vol. 5, no. 1, pp. 67–76, March 2001.