Logistics Management
Location Strategy

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Location Strategy

- What's located?
- Sourcing points
  - Plants
  - Vendors
  - Ports
- Intermediate points
  - Warehouses
  - Terminals
  - Public facilities (fire, police, and ambulance stations)
  - Service centers
- Sink points
  - Retail outlets
  - Customers/Users

Location Strategy

- Key Questions
  - How many facilities should there be?
  - Where should they be located?
  - What size should they be?

- Why Location is Important?
  - Gives structure to the network
  - Significantly affects inventory and transportation costs
  - Impacts on the level of customer service to be achieved

Location Decisions

- Single Facility Location
- Multiple Facility Location
- Retail/service Location

Nature of Location Analysis

- Manufacturing (plants & warehouses)
  - Decisions are driven by economics. Relevant costs such as transportation, inventory carrying, labor, and taxes are traded off against each other to find good locations.
- Retail
  - Decisions are driven by revenue. Traffic flow and resulting revenue are primary location factors, cost is considered after revenue.
- Service
  - Decisions are driven by service factors. Response time, accessibility, and availability are key dimensions for locating in the service industry.

Single Facility Location

- Locating a single plant, terminal, warehouse, or retail or service point.

- Center-of-Gravity (COG) method
  - A continuous location method
  - Locates on the basis of transportation costs alone

- The COG method involves
  - Determining the volumes by source and destination point
  - Determining the transportation costs based on $/unit/mi.
  - Overlaying a grid to determine the coordinates of source and/or destination points
  - Finding the weighted center of gravity for the graph

COG Method

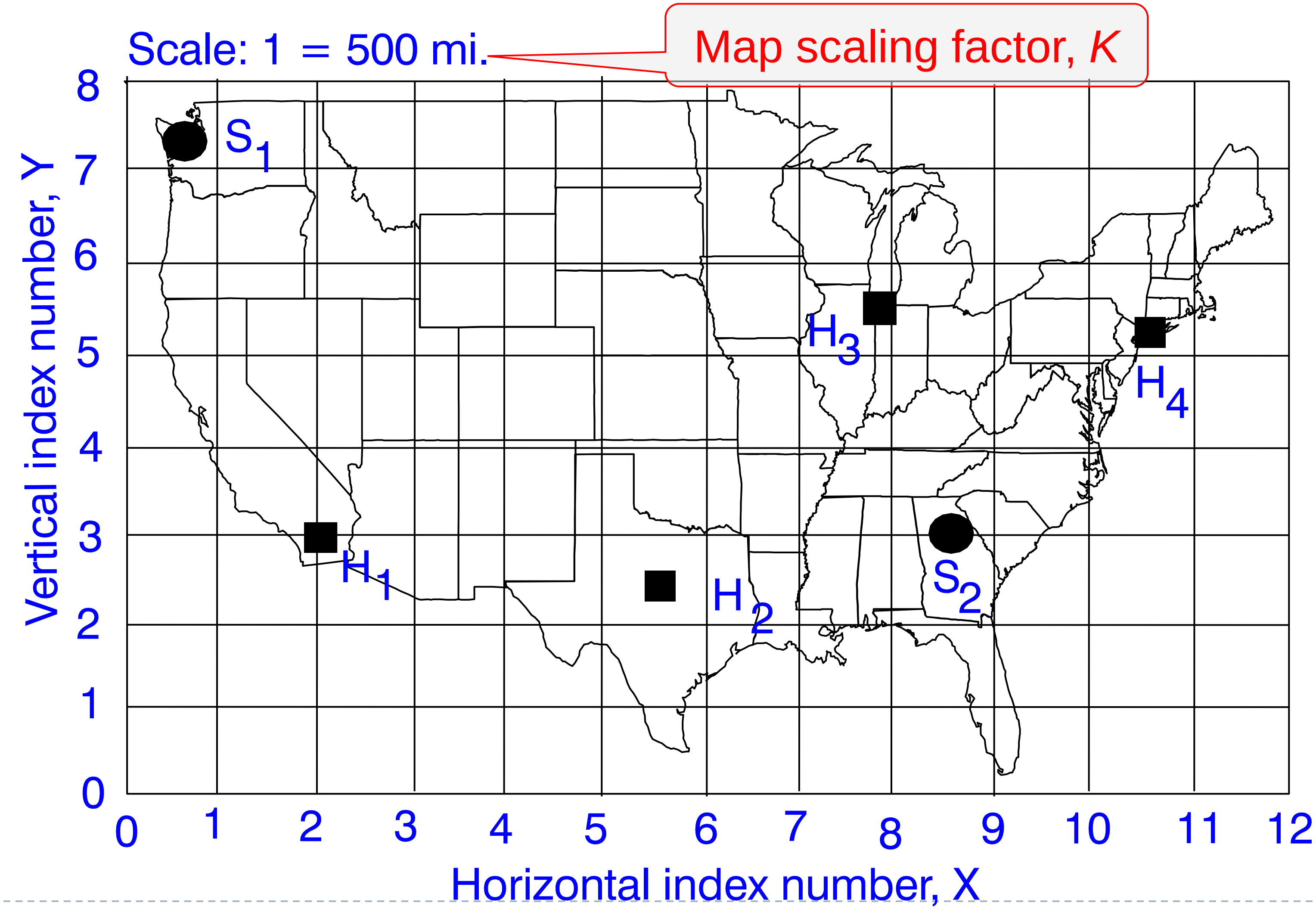
- $Min\ TC = \sum_i V_i R_i d_i$
  - TC = total transportation cost
  - V_i = volume at point i
  - R_i = transportation rate to point i
  - d_i = distance to point i from the facility to be located
- The facility Location:
  - $\bar{X} = \frac{\sum_i V_i R_i X_i}{\sum_i V_i R_i}$, $\bar{Y} = \frac{\sum_i V_i R_i Y_i}{\sum_i V_i R_i}$,
  - X_i, Y_i = coordinate points for point i
  - $\bar{X}$, $\bar{Y}$ = coordinate points for facility to be located

COG Method Example

- Suppose a regional medical warehouse is to be established to serve several Veterans Administration hospitals throughout the country. The supplies originate at S_1 and S_2 and are destined for hospitals at H_1 through H_4. The relative locations are shown on the map grid. Other data are:

Point i	Products	Location	Annual Volume, cwt.	Rate $/cwt/mi.	X_i	Y_i
1 S_1	A	Seattle	8,000	0.02	0.6	7.3
2 S_2	B	Atlanta	10,000	0.02	8.6	3.0
3 H_1	A&B	Los Angeles	5,000	0.05	2.0	3.0
4 H_2	A&B	Dallas	3,000	0.05	5.5	2.4
5 H_3	A&B	Chigago	4,000	0.05	7.9	5.5
6 H_4	A&B	New York	6,000	0.05	10.6	5.2

COG Method Example



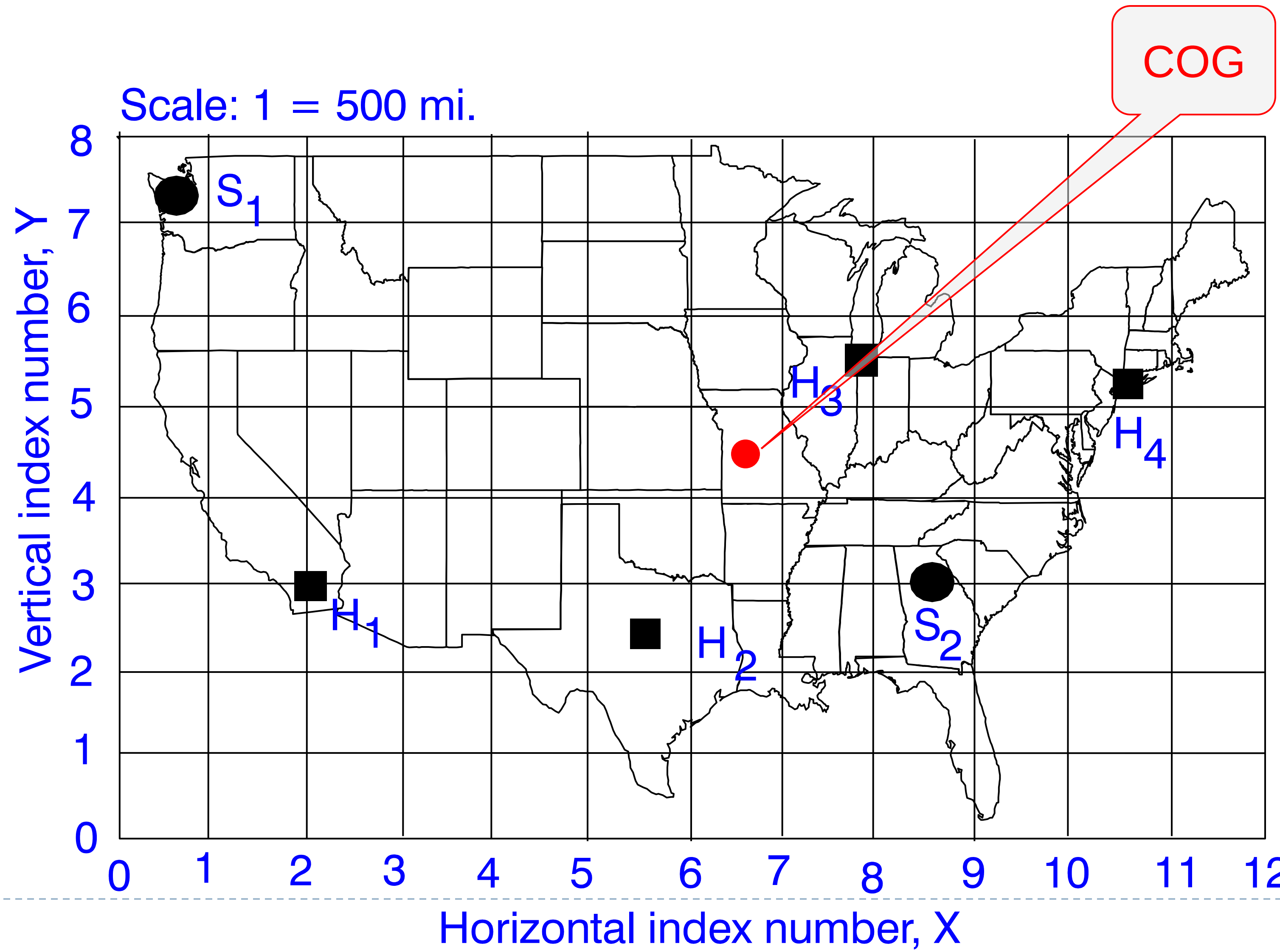
COG Method Example

i	X_i	Y_i	V_i	R_i	V_iR_i	$V_iR_iX_i$	$V_iR_iY_i$
1	0.6	7.3	8000	0.02	160	96	1168
2	8.6	3	10000	0.02	200	1720	600
3	2	3	5000	0.05	250	500	750
4	5.5	2.4	3000	0.05	150	825	360
5	7.9	5.5	4000	0.05	200	1580	1100
6	10.6	5.2	6000	0.05	300	3180	1560
					1260	**7901**	**5538**

$\bar{X}$= 7,901/1,260 = 6.27

$\bar{Y}$= 5,538/1,260 = 4.40

COG Method Example



COG Method Example

- The total cost for this location is found by:

$$TC = \sum_i V_i R_i K \sqrt{(X_i - \bar{X})^2 + (Y_i - \bar{Y})^2}$$

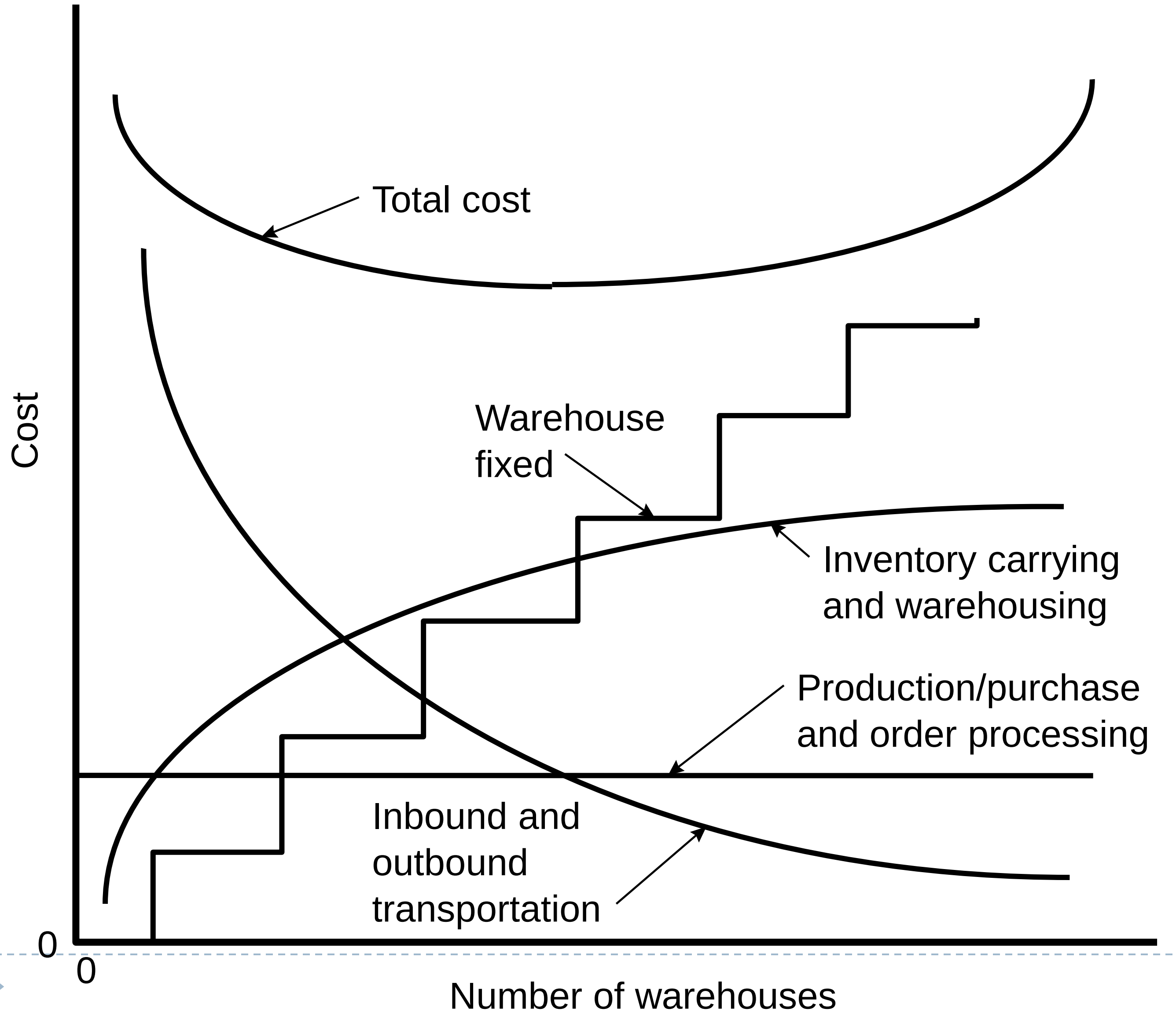
  - K is the map scaling factor to convert coordinates into miles.

i	X_i	Y_i	V_i	R_i	TC
1	0.6	7.3	8000	0.02	509,706
2	8.6	3	10000	0.02	271,526
3	2	3	5000	0.05	561,597
4	5.5	2.4	3000	0.05	160,417
5	7.9	5.5	4000	0.05	196,859
6	10.6	5.2	6000	0.05	660,529
					2,360,633

Multiple Location Methods

- A more complex problem that most firms have.
- It involves trading off the following costs:
  - Transportation inbound to and outbound from the facilities
  - Storage and handling costs
  - Inventory carrying costs
  - Production/purchase costs
  - Facility fixed costs
- Subject to:
  - Customer service constraints
  - Facility capacity restrictions
- Mathematical methods are popular for this type of problem that
  - Search for the best combination of facilities to minimize costs
  - Do so within a reasonable computational time
  - Do not require enormous amounts of data for the analysis

Location Cost Trade-Offs



Examples of Practical COG Model Use

- Location of truck maintenance terminals
- Location of public facilities such as offices, and police and fire stations
- Location of medical facilities
- Location of most any facility where transportation cost (rather than inventory carrying cost and facility fixed cost) is the driving factor in location
- As a suggestor of sites for further evaluation

Multiple COG

- Formulated as basic COG model
- Can search for the best locations for a selected number of sites.
- Fixed costs and inventory consolidation effects are handled outside of the model.

- A multiple COG procedure
  - Rank demand points from highest to lowest volume
  - Use the M largest as initial facility locations and assign remaining demand centers to these locations
  - Compute the COG of the M locations
  - Reassign all demand centers to the M COGs on the basis of proximity
  - Recompute the COGs and repeat the demand center assignments, stopping this iterative process when there is no further change in the assignments or COGs

Multiple COG Example

- Warehouse Cost = $800{,}000\sqrt{N}$
- For N = 1
- Total cost = Transportation cost + Warehouse Cost
  - 2,360,633 + 800,000 = 3,160,633

i	X_i	Y_i	V_i	R_i	TC
1	0.6	7.3	8000	0.02	509,706
2	8.6	3	10000	0.02	271,526
3	2	3	5000	0.05	561,597
4	5.5	2.4	3000	0.05	160,417
5	7.9	5.5	4000	0.05	196,859
6	10.6	5.2	6000	0.05	660,529
					2,360,633

Multiple COG Example

- For N = 2
- Determine initial locations
- $w_1(8.6, 3)$ -- $w_2(0.6, 7.3)$

i	X_i	Y_i	V_i
1	0.6	7.3	8000
2	8.6	3	10000
3	2	3	5000
4	5.5	2.4	3000
5	7.9	5.5	4000
6	10.6	5.2	6000

- Compute the distance of each point from initial locations
- Determine the cluster of each point

i	X_i	Y_i	d_1	d_2	Cluster #
1	0.6	7.3	9.08	0.00	2
2	8.6	3	0.00	9.08	1
3	2	3	6.60	4.52	2
4	5.5	2.4	3.16	6.93	1
5	7.9	5.5	2.60	7.52	1
6	10.6	5.2	2.97	10.22	1

Multiple COG Example

- COG for the first Cluster

i	X_i	Y_i	V_i	R_i	V_iR_i	V_iX_i	V_iY_i
2	8.6	3	10000	0.02	200	1720	600
4	5.5	2.4	3000	0.05	150	825	360
5	7.9	5.5	4000	0.05	200	1580	1100
6	10.6	5.2	6000	0.05	300	3180	1560
					850	**7305**	**3520**

w_1= (7305/850; 3520/850)= (8.59,4.26)

- COG for the second Cluster

i	X_i	Y_i	V_i	R_i	V_iR_i	V_iX_i	V_iY_i
1	0.6	7.3	8000	0.02	160	96	1168
3	2	3	5000	0.05	250	500	750
					410	**596**	**1918**

w_2= (596/410; 1918/410)= (1.45, 4.68)

Multiple COG Example

- For $w_1(8.59,4.26)$ – $w_2(1.45, 4.68)$
- Compute the distance of each point from locations
- Determine new clusters of each point

i	X_i	Y_i	d_1	d_2	Cluster #
1	0.6	7.3	8.55	2.76	2
2	8.6	3	1.26	7.34	1
3	2	3	6.71	1.76	2
4	5.5	2.4	3.61	4.64	1
5	7.9	5.5	1.42	6.50	1
6	10.6	5.2	2.22	9.16	1

- Clusters do not change, stop procedure!

Multiple COG Example

- Calculate Transportation cost for N =2

i	Xi	Yi	Vi	Rj	wx	wy	Distance	Transportation Cost
1	0.6	7.3	8000	0.02	1.45	4.68	2.76	220,594
2	8.6	3	10000	0.02	8.59	4.26	1.26	125,884
3	2	3	5000	0.05	1.45	4.68	1.76	220,594
4	5.5	2.4	3000	0.05	8.59	4.26	3.61	270,716
5	7.9	5.5	4000	0.05	8.59	4.26	1.42	142,208
6	10.6	5.2	6000	0.05	8.59	4.26	2.22	332,357
								1,312,351

- Total Cost (N=2) = 1,312,351 + 800.000$\sqrt{2}$= 2,443,722
- Total Cost (N=1) = 2,360,633 + 800,000 = 3,160,633

Multiple COG Example

- Minimum cost at N = 3;

i	Xi	Yi	Vi	Rj	wx	wy	Cluster #	Distance	Transportation Cost
1	0.6	7.3	8000	0.02	1.45	4.68	2	2.76	220,594
2	8.6	3	10000	0.02	7.50	3.75	1	1.33	132,880
3	2	3	5000	0.05	1.45	4.68	2	1.76	220,594
4	5.5	2.4	3000	0.05	7.50	3.75	1	2.41	180,783
5	7.9	5.5	4000	0.05	7.50	3.75	1	1.80	179,956
6	10.6	5.2	6000	0.05	10.60	5.20	3	0.00	0
									934,807

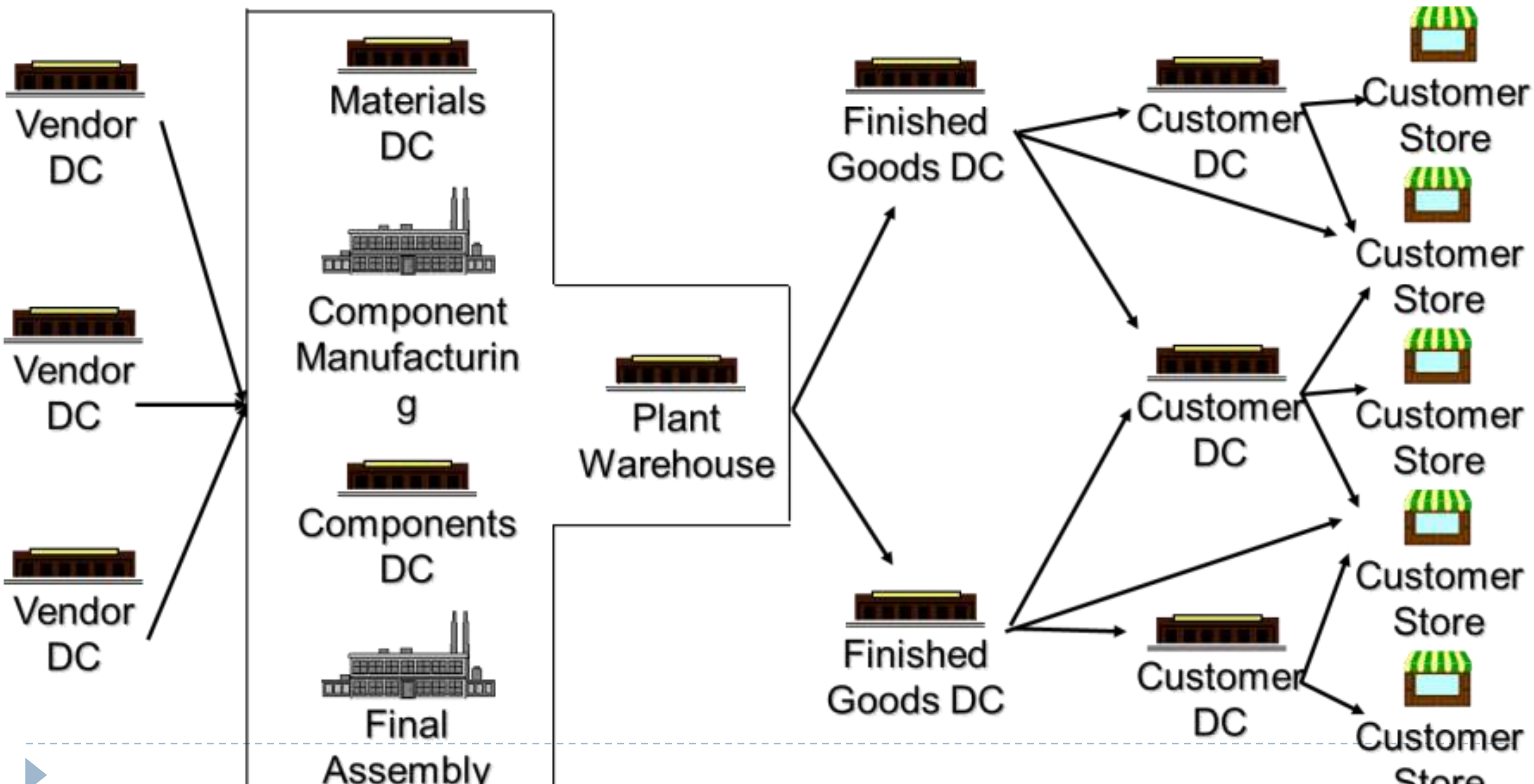
Total Cost (N=3) = 934,807 + 800.000$\sqrt{3}$ = 2,320,447

Total Cost (N=2) = 1,312,351 + 800.000$\sqrt{2}$= 2,443,722

Total Cost (N=1) = 2,360,633 + 800,000 = 3,160,633

Multifacility Location Models
Places are Already Known

- Conventional Network

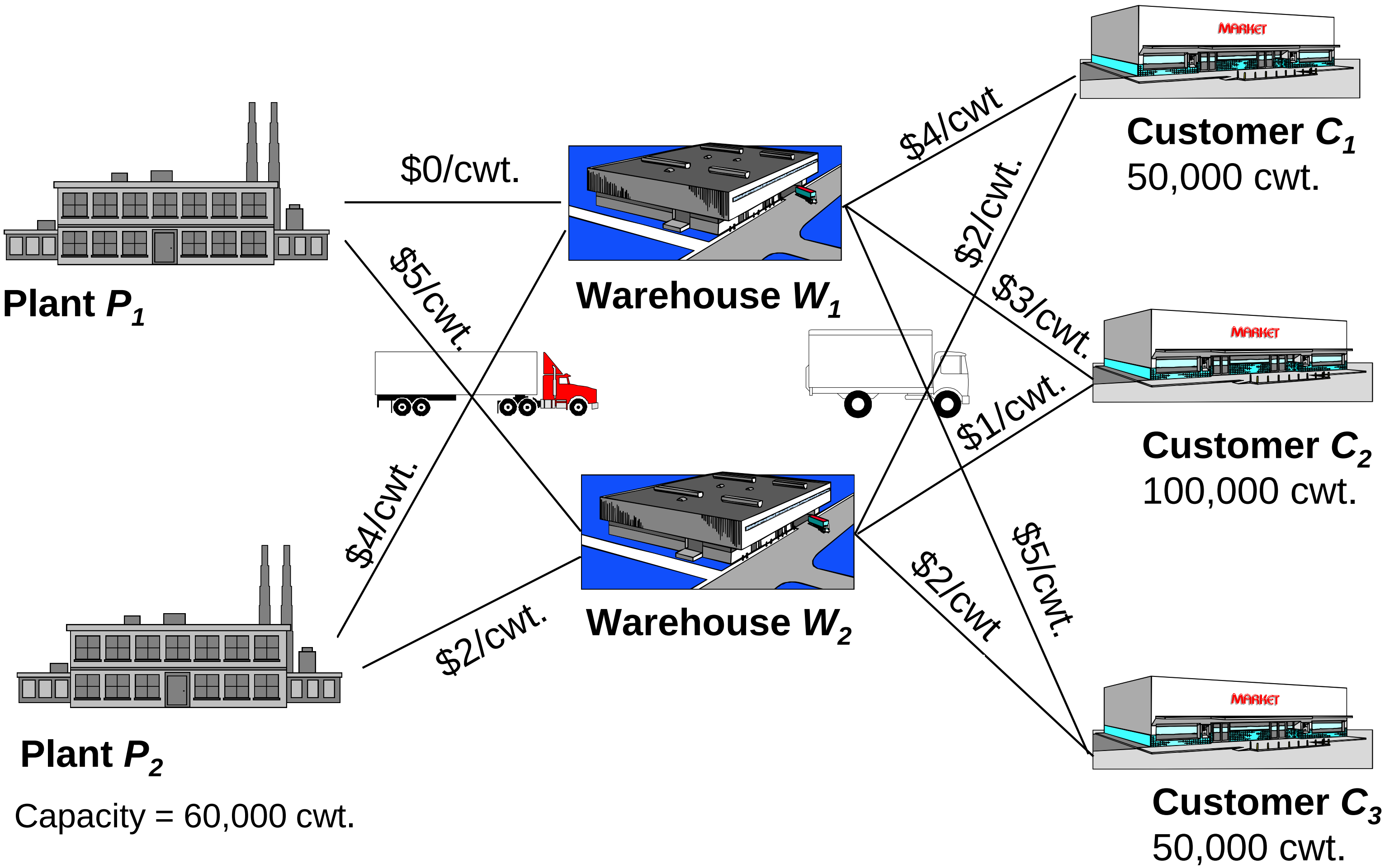


Multifacility Location Models
Places are Already Known

- Consider the following distribution system:
  - Single product
  - Two plants p_1, p_2
  - Plant p_2 has an annual capacity of 60,000 units
  - The two plants have the same production costs
  - Two existing warehouses, referred to as warehouse w_1 and warehouse w_2 have identical warehouse handling costs
  - Three markets, c_1, c_2, c_3 with demands of 50,000, 100,000 and 50,000 respectively

Facility Warehouse	p_1	p_2	c_1	c_2	c_3
w_1	0	4	3	4	5
w_2	5	2	2	1	2

Multifacility Location Models
Places are Already Known



Multifacility Location Models Heuristics

- Heuristic 1
- For each market we choose the cheapest warehouse to source demand.
  - c_1, c_2, c_3 would be supplied by w_2.
  - For this warehouse choose the cheapest plant;
    - 60,000 units from p_2
    - the remaining 140,000 from p_1.
  - Total cost = 2*50,000 + 1*100,000 + 2*50,000 + 2*60,000 + 5*140,000 = 1,120,000

Multifacility Location Models Heuristics

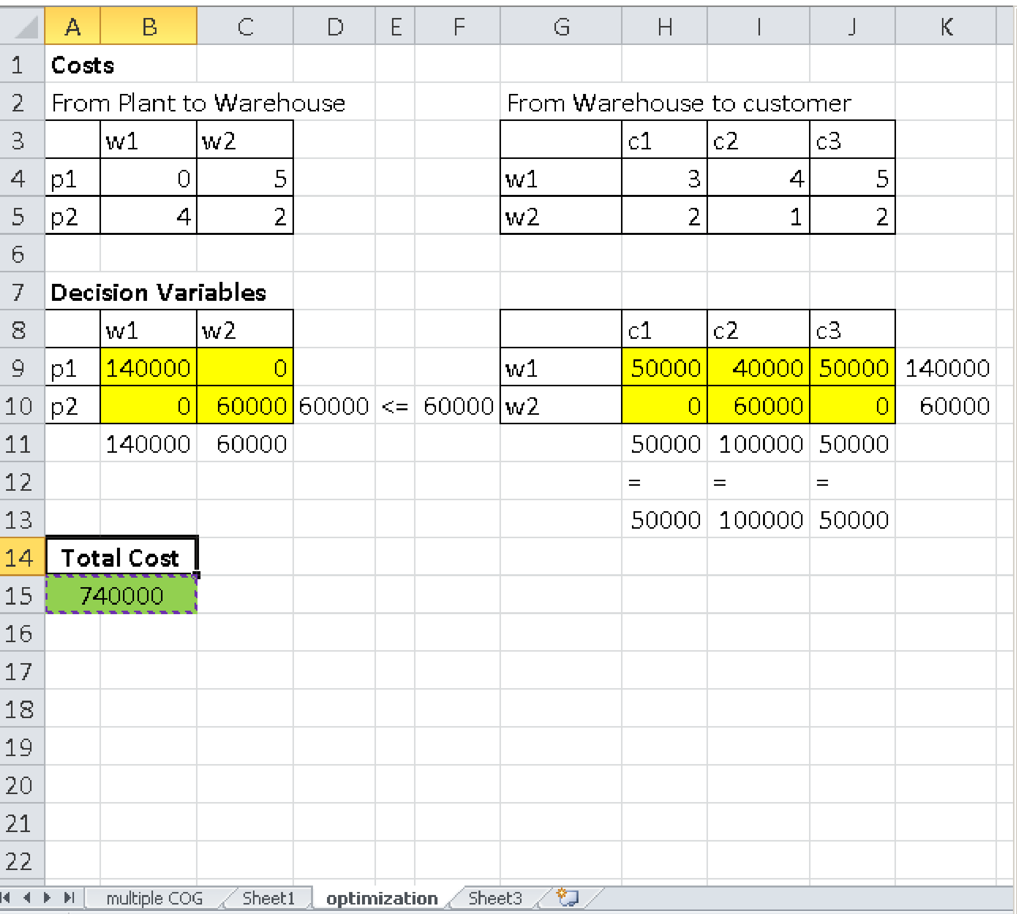
- Heuristic 2
- For each market area, choose the warehouse where the total delivery costs to and from the warehouse are the lowest; that is, consider inbound and outbound distribution costs.
  - Thus for market area c_1, consider the paths
    - p_1-w_1-c_1, p_1-w_2-c_1, p_2-w_1-c_1, p_2-w_2-c_1.
    - The cheapest is p_1-w_1-c_1, so choose w_1 for c_1.
  - using a similar analysis, we choose w_2 for c_2 and w_2 for c_3.
  - This implies that warehouse w1 delivers a total of 50,000 units while warehouse w2 delivers a total of 150,000 units.
  - The best inbound flow pattern is to supply 50,000 from plant p1 to warehouse w1, supply 60,000 units from plant p2 to warehouse w2, and supply 90,000 from plant p1 to warehouse w2.
  - The total cost for this strategy is 920,000.

Multifacility Location Models
Optimization Model

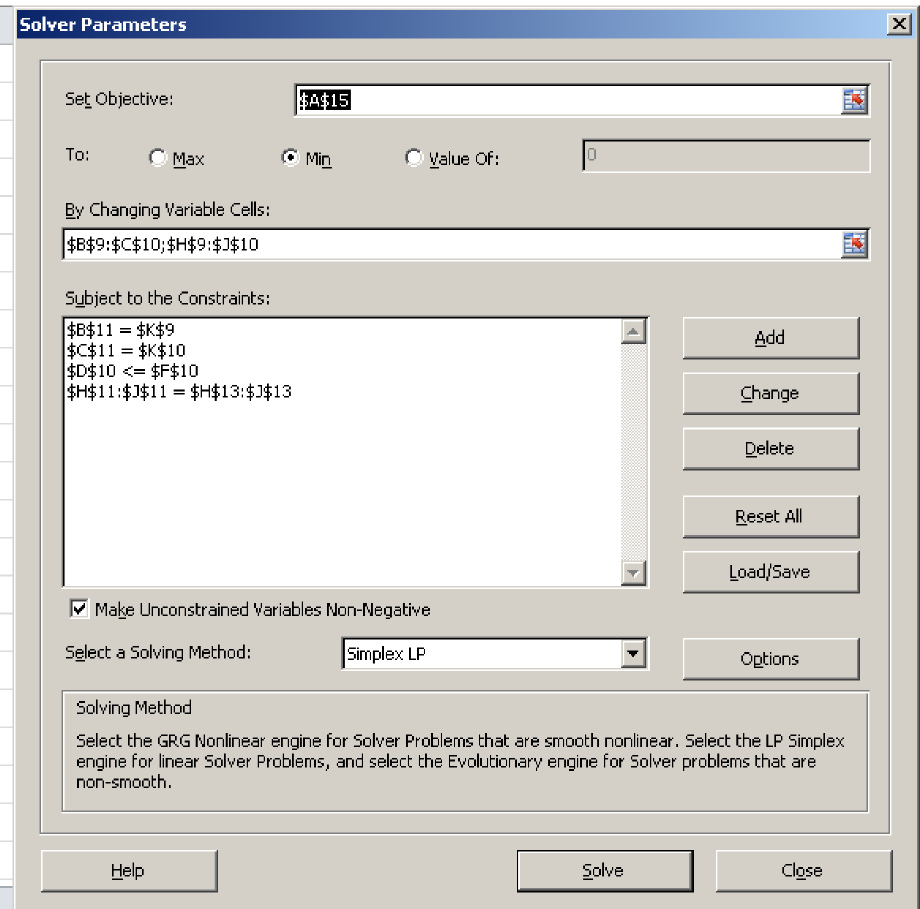
- Places are already known
- Minimize total transportation cost
  - $0X(p_1,w_1)+5X(p_1,w_2)+4X(p_2,w_1)+2X(p_2,w_2)$
  - $+3X(w_1,c_1)+4X(w_1,c_2)+5X(w_1,c_3)+2X(w_2,c_1)+1X(w_2,c_2)+2X(w_2,c_3)$
- s.t.
  - $X(p_2,w_1)+X(p_2,w_2)\leq 60,000$ Plant 2 capacity
  - $X(p_1,w_1)+X(p_2,w_1)=X(w_1,c_1)+X(w_1,c_2)+X(w_1,c_3)$ Whs.1 input/output
  - $X(p_1,w_2)+X(p_2,w_2)=X(w_2,c_1)+X(w_2,c_2)+X(w_2,c_3)$ Whs.2 input/output
  - $X(w_1,c_1)+X(w_2,c_1)=50,000$ Customer 1 demand
  - $X(w_1,c_2)+X(w_2,c_2)=100,000$ Customer 2 demand
  - $X(w_1,c_3)+X(w_2,c_3)=50,000$ Customer 3 demand

Multifacility Location Models
Optimization Model

- EXCEL Solver



	A	B	C	D	E	F	G	H	I	J	K
1	Costs										
2	From Plant to Warehouse						From Warehouse to customer				
3		w1	w2					c1	c2	c3	
4	p1	0	5				w1	3	4	5	
5	p2	4	2				w2	2	1	2	
6											
7	Decision Variables										
8		w1	w2					c1	c2	c3	
9	p1	140000	0				w1	50000	40000	50000	140000
10	p2	0	60000	60000	<=	60000	w2	0	60000	0	60000
11		140000	60000					50000	100000	50000	
12								=	=	=	
13								50000	100000	50000	
14	Total Cost										
15	740000										



Multifacility Location Models
Optimization Model

- Result
- Total Cost: $ 740,000

	w_1	w_2		c_1	c_2	c_3
p_1	140000	0	w_1	50000	40000	50000
p_2	0	60000	w_2	0	60000	0

Retail Location

- Contrasts with plant and warehouse location.
- Factors other than costs such as parking, nearness to competitive outlets, and nearness to customers are dominant

- Methods
- Weighted checklist
  - Often many of the factors that are important to retail location are not easily or inexpensively quantified
  - Judgment is an integral part of the decision
  - Good where many subjective factors are involved
  - Quantifies the comparison among alternate locations
- Spatial-Interaction Model
  - The gravity model to determining the drawing power, or overall desirability, of a site
  - The basic idea is that two competing cities attract trade from an intervening town in direct propotion to each city's population but inverse proportion to square distance between cities and town.

A Hypothetical Weighted Factor Checklist for a Retail Location Example

Factor Weight (1 to 10)	Location Factors	Factor Score (1 to 10)	Weighted Score
8	Proximity to competing stores	5	40
5	Space rent/lease considerations	3	15
8	Parking space	10	80
7	Proximity to complementary stores	8	56
6	Modernity of store space	9	54
9	Customer accessibility	8	72
3	Local taxes	2	6
3	Community service	4	12
8	Proximity to major transportation arteries	7	56
	Total Index		**391**

Factor weights approaching 10 indicate great importance.
Factor scores approaching 10 refer to a favored location status.

Spatial-Interaction Model
Huff's Gravity Model

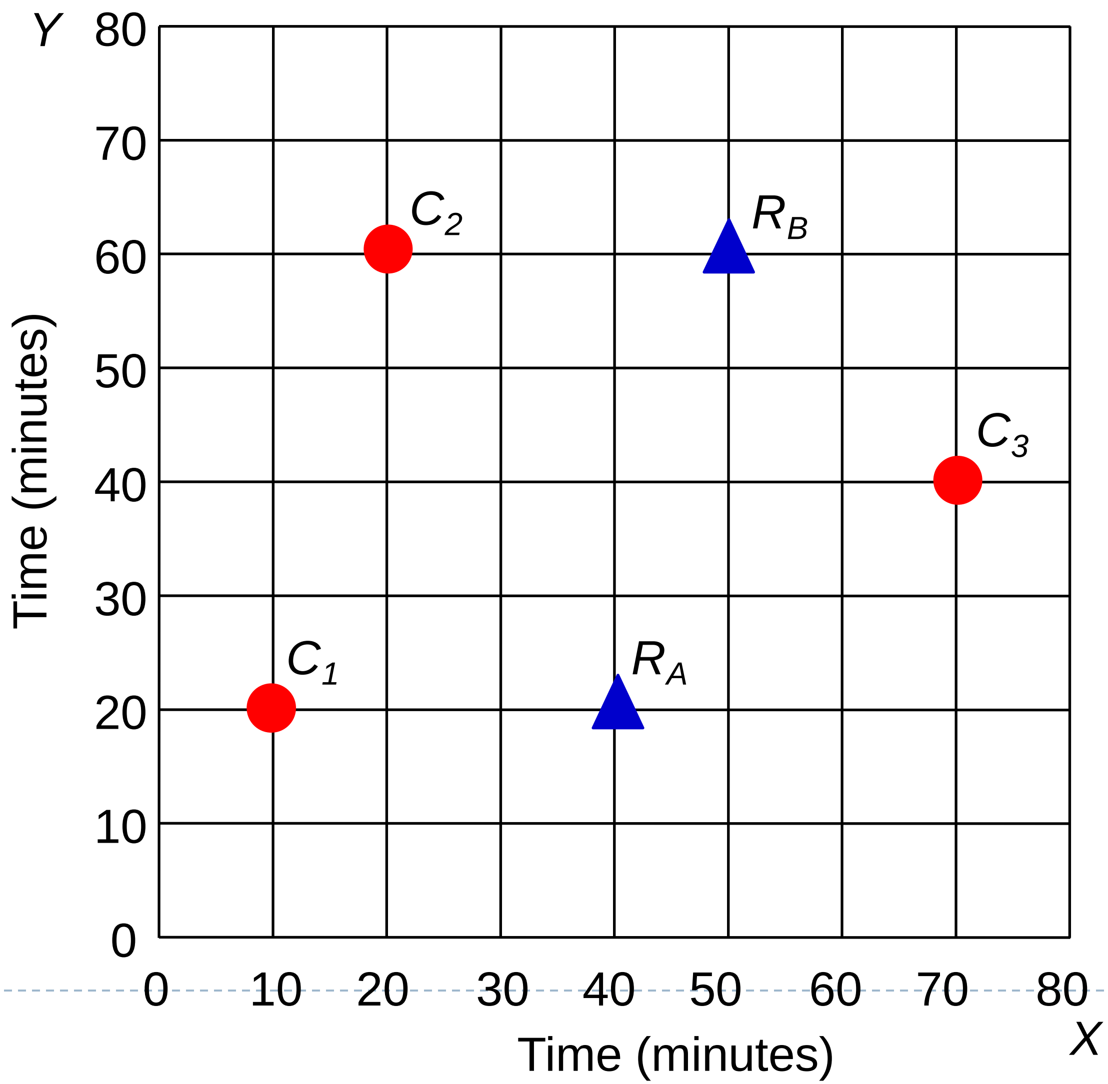
- A take-off on Newton's law of gravity.
- "Mass" or retail "variety" attracts customers, and the distance from customer repels them.
- The basic model
- $$E_{ij} = P_{ij}C_i = \frac{S_j/T_{ij}^a}{\sum_j S_j/T_{ij}^a} C_i$$
  - E_{ij} = expected demand from population center *i* that will be attracted to retail location *j*.
  - P_{ij} = probability of customers from *i* travelling to retail location *j*.
  - C_i = customer demand at point *i*
  - S_j = size of retail location *j*
  - T_{ij} = travel time between customer location *i* and retail location *j*
  - *n* = number of retail locations *j*
  - *a* = empirically estimated parameter

Huff's Gravity Model Example

- Two shopping centers (R_A and R_B) are to attract customers from C_1, C_2, and C_3. Shopping center A has 500,000 square feet of selling area whereas center B has 1,000,000. The customer clusters have a buying potential of \$10, \$5, and \$7 million, respectively. The parameter *a* is estimated to be 2. What is the sales potential of each shopping center?

Customer	Time from Customer *i* to Location *j*		T_{ij}^2		S_j/T_{ij}^2		P_{ij}		Potential	$E_{ij} = P_{ij}C_{ij}$	
	A	B	A	B	A	B	A	B		A	B
C_1	30	56,6	900	3204	556	312	0,640	0,360	10	6,403	3,597
C_2	44,7	30	1998	900	250	1111	0,184	0,816	5	0,919	4,081
C_3	36	28,3	1296	801	386	1249	0,236	0,764	7	1,652	5,348

Huff's Gravity Model Example



Other Methods for Retail Location

- Regression Analysis (to forecast the revenues that a specific site can expect
- Covering models (particularly useful for locating emergency services such as police and fire stations)
- Game Theory (suggested when competition is a key factor)
- Location-Allocation models such as goal programming and integer programming (see example at the blackboard)

Next Class

- Final Exam

- June 20, 2012
- The exam will be in room D301 at 19:30.

- All course topics are included in the exam.

- It is strictly forbidden to use mobile phones for calculations or other purposes.
- Please provide calculator for calculations.

- No class on June 13!