Logistics Management
Inventory – Cycle Inventory

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Role of Inventory in the Supply Chain

- Improve Matching of Supply and Demand
- Improved Forecasting
- Reduce Material Flow Time
- Reduce Waiting Time
- Reduce Buffer Inventory

- Economies of Scale
- Supply / Demand Variability
- Seasonal Variability
- Cycle Inventory
- Safety Inventory
- Seasonal Inventory
What are Inventories?

- Finished product held for sale
- Goods in warehouses
- Work in process
- Goods in transit
- Staff hired to meet service needs
- Any owned or financially controlled raw material, work in process, and/or finished good or service held in anticipation of a sale but not yet sold
Where are Inventories?

Inventory locations

Material sources

Inbound transportation

Production

Outbound transportation

Finished goods warehousing

Customers

Receiving

Production materials

Inventories in-process

Finished goods

Shipping
Reasons for Inventories

- Improve customer service
  - Provides immediacy in product availability
- Encourage production, purchase, and transportation economies
  - Allows for long production runs
  - Takes advantage of price-quantity discounts
  - Allows for transport economies from larger shipment sizes
- Act as a hedge against price changes
  - Allows purchasing to take place under most favorable price terms
- Protect against uncertainties in demand and lead times
  - Provides a measure of safety to keep operations running when demand levels and lead times cannot be known for sure
- Act as a hedge against contingencies
  - Buffers against such events as strikes, fires, and disruptions in supply
Reasons Against Inventories

- They consume capital resources that might be put to better use elsewhere in the firm.

- They too often mask quality problems that would more immediately be solved without their presence.

- They divert management’s attention away from careful planning and control of the supply and distribution channels by promoting an insular attitude about channel management.
Types of Inventories

- **Pipeline**
  - Inventories in transit

- **Speculative**
  - Goods purchased in anticipation of price increases

- **Regular/Cyclical/Seasonal**
  - Inventories held to meet normal operating needs

- **Safety**
  - Extra stocks held in anticipation of demand and lead time uncertainties

- **Obsolete/Dead Stock**
  - Inventories that are of little or no value due to being out of date, spoiled, damaged, etc.
Costs Relevant to Inventory Management

- **Carrying costs**
  - Cost for holding the inventory over time
  - The primary cost is the cost of money tied up in inventory, but also includes obsolescence, insurance, personal property taxes, and storage costs
  - Typically, costs range from the cost of short term capital to about 40%/year. The average is about 25%/year of the item value in inventory.

- **Procurement costs**
  - Cost of preparing the order
  - Cost of order transmission
  - Cost of production setup if appropriate
  - Cost of materials handling or processing at the receiving dock
  - Price of the goods
Costs Relevant to Inventory Management

- **Out-of-stock costs**
  - Lost sales cost
    - Profit immediately foregone
    - Future profits foregone through loss of goodwill
  - Backorder cost
    - Costs of extra order handling
    - Additional transportation and handling costs
    - Possibly additional setup costs
Good inventory management is a careful balancing act between stock availability and the cost of holding inventory.

- **Service objectives**
  - Setting stocking levels so that there is only a specified probability of running out of stock

- **Cost objectives**
  - Balancing conflicting costs to find the most economical replenishment quantities and timing
Managing Economies of Scale in the Supply Chain: Cycle Inventory

- Role of Cycle Inventory in a Supply Chain
- Economies of Scale to Exploit Fixed Costs
- Economies of Scale to Exploit Quantity Discounts
- Short-Term Discounting: Trade Promotions
Role of Cycle Inventory in a Supply Chain

- **Lot, or batch size**: quantity that a supply chain stage either produces or orders at a given time.

- **Cycle inventory**: average inventory that builds up in the supply chain because a supply chain stage either produces or purchases in lots that are larger than those demanded by the customer.
  - \( Q \) = lot or batch size of an order
  - \( D \) = demand per unit time

- **Inventory profile**: plot of the inventory level over time.
  - **Cycle inventory** \( = Q/2 \) (depends directly on lot size)

- **Average flow time** \( = \text{Avg inventory} / \text{Avg flow rate} \)
  - **Average flow time from cycle inventory** \( = Q/(2D) \)
Reorder Point Method Under Certainty for a Single Item

Inventory Level

Reorder point, $R$

$Q$

Order Placed

Order Received

Lead time

Order Placed

Order Received

Lead time

Time

Quantity on-hand $plus$ on-order
Role of Cycle Inventory in a Supply Chain

\[ Q = 1000 \text{ units} \]
\[ D = 100 \text{ units/day} \]

Cycle inventory = \( \frac{Q}{2} = \frac{1000}{2} = 500 \) = Avg inventory level from cycle inventory

\[ \text{Avg flow time} = \frac{Q}{2D} = \frac{1000}{(2)(100)} = 5 \text{ days} \]

- Cycle inventory adds 5 days to the time a unit spends in the supply chain

- Lower cycle inventory is better because:
  - Average flow time is lower
  - Working capital requirements are lower
  - Lower inventory holding costs
Role of Cycle Inventory in a Supply Chain

- Cycle inventory is held primarily to take advantage of economies of scale in the supply chain
- Supply chain costs influenced by lot size:
  - Material cost = $C$
  - Fixed ordering cost = $S$
  - Holding cost = $H = hC$ ($h$ = cost of holding $1$ in inventory for one year)
- Primary role of cycle inventory is to allow different stages to purchase product in lot sizes that minimize the sum of material, ordering, and holding costs
- Ideally, cycle inventory decisions should consider costs across the entire supply chain, but in practice, each stage generally makes its own supply chain decisions – increases total cycle inventory and total costs in the supply chain
Estimating Cycle Inventory Related Costs in Practice

- **Inventory Holding Cost**
  - Obsolescence
  - Handling costs
  - Occupancy costs
  - Theft, security, damage, tax, insurance

- **Ordering Cost**
  - Buyer time
  - Transportation costs
  - Receiving costs
  - Unique other costs
Economies of Scale to Exploit Fixed Costs

- How do you decide whether to go shopping at a convenience store or at Sam’s Club?
- Lot sizing for a single product (EOQ)
- Aggregating multiple products in a single order
- Lot sizing with multiple products or customers
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all products
  - Lots are ordered and delivered jointly for a subset of products
Economies of Scale
to Exploit Fixed Costs

Annual demand = D
Number of orders per year = D/Q
Annual material cost = CD
Annual order cost = (D/Q)S
Annual holding cost = (Q/2)H = (Q/2)hC
Total annual cost = TC = CD + (D/Q)S + (Q/2)hC

Figure 10.2 shows variation in different costs for different lot sizes at Best Buy
Inventory’s Conflicting Cost Patterns

- Material cost
- Ordering cost
- Holding cost
- Total cost

EOQ (Economic Order Quantity)
Fixed Costs: Optimal Lot Size and Reorder Interval (EOQ)

D: Annual demand
S: Setup or Order Cost
C: Cost per unit
h: Holding cost per year as a fraction of product cost
H: Holding cost per unit per year
Q: Lot Size, Q*: Optimal Lot Size
n*: Optimal order frequency

Material cost is constant and therefore is not considered in this model

\[ H = hC \]
\[ Q^* = \sqrt{\frac{2DS}{H}} \]
\[ n^* = \sqrt{\frac{DhC}{2S}} \]
Example - EOQ

Demand, $D = 12,000$ computers per year
Unit cost per lot, $C = $500
Holding cost per year as a fraction of unit cost, $h = 0.2$
Fixed cost, $S = $4,000/order

$Q^* = \sqrt{\frac{2 \times 12000 \times 4000}{0.2 \times 500}} = 980$ computers
Cycle inventory $= \frac{Q^*}{2} = 490$

Average Flow time $= \frac{Q^*}{2D} = \frac{980}{2 \times 12000} = 0.041$ year $= 0.49$ month

$n^* = \sqrt{\frac{12000 \times 0.2 \times 500}{2 \times 4000}} = 12.24$ orders
Example - EOQ (continued)

Annual ordering and holding cost =
= \frac{12000}{980} \times 4000 + \frac{980}{2} \times 0.2 \times 500 = $97,980

Suppose lot size is reduced to Q=200, which would reduce flow time:

Annual ordering and holding cost =
= \frac{12000}{200} \times 4000 + \frac{200}{2} \times 0.2 \times 500 = $250,000

To make it economically feasible to reduce lot size, the fixed cost associated with each lot would have to be reduced.
Example – Relationship between desired lot size and ordering cost

If desired lot size = \( Q^* = 200 \) units, what would \( S \) have to be?

\[ D = 12000 \text{ units} \]
\[ C = $500 \]
\[ h = 0.2 \]

Use EOQ equation and solve for \( S \):

\[ S = \frac{hC(Q^*)^2}{2D} = \frac{(0.2)(500)(200)^2}{(2)(12000)} = $166.67 \]

To reduce optimal lot size by a factor of \( k \), the fixed order cost must be reduced by a factor of \( k^2 \)
Key Points from EOQ Model

- In deciding the optimal lot size, the tradeoff is between setup (order) cost and holding cost.

- If demand increases by a factor of 4, it is optimal to increase batch size by a factor of 2 and produce (order) twice as often. *Cycle inventory (in days of demand) should decrease as demand increases.*

- If lot size is to be reduced, one has to reduce fixed order cost. To reduce lot size by a factor of 2, order cost has to be reduced by a factor of 4.
Aggregating Multiple Products in a Single Order

- Transportation is a significant contributor to the fixed cost per order
- Can possibly combine shipments of different products from the same supplier
  - same overall fixed cost
  - shared over more than one product
  - effective fixed cost is reduced for each product
  - lot size for each product can be reduced
- Can also have a single delivery coming from multiple suppliers or a single truck delivering to multiple retailers
- Aggregating across products, retailers, or suppliers in a single order allows for a reduction in lot size for individual products because fixed ordering and transportation costs are now spread across multiple products, retailers, or suppliers
Example: Aggregating Multiple Products in a Single Order

- Suppose there are 4 computer products in the previous example: Deskpro, Litepro, Medpro, and Heavpro.
- Assume demand for each is 1000 units per month.
- If each product is ordered separately:
  - $Q^* = 980$ units for each product.
  - Total cycle inventory = $4(Q/2) = (4)(980)/2 = 1960$ units.
- Aggregate orders of all four products:
  - Combined $Q^* = 1960$ units.
  - For each product: $Q^* = 1960/4 = 490$.
  - Cycle inventory for each product is reduced to $490/2 = 245$.
  - Total cycle inventory = $1960/2 = 980$ units.
  - Average flow time, inventory holding costs will be reduced.
Lot Sizing with Multiple Products or Customers

In practice, the fixed ordering cost is dependent at least in part on the variety associated with an order of multiple models.

- A portion of the cost is related to transportation (independent of variety).
- A portion of the cost is related to loading and receiving (not independent of variety).

Three scenarios:

- Lots are ordered and delivered independently for each product.
- Lots are ordered and delivered jointly for all three models.
- Lots are ordered and delivered jointly for a selected subset of models.
Lot Sizing with Multiple Products

- Demand per year
  - $D_L = 12,000; D_M = 1,200; D_H = 120$
- Common transportation cost, $S = $4,000
- Product specific order cost
  - $s_L = $1,000; s_M = $1,000; s_H = $1,000$
- Holding cost, $h = 0.2$
- Unit cost
  - $C_L = $500; C_M = $500; C_H = $500$
Delivery Options

- No Aggregation: Each product ordered separately
- Complete Aggregation: All products delivered on each truck
- Tailored Aggregation: Selected subsets of products on each truck
## No Aggregation: Order Each Product Independently

<table>
<thead>
<tr>
<th></th>
<th>Litepro</th>
<th>Medpro</th>
<th>Heavypro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand per year</td>
<td>12,000</td>
<td>1,200</td>
<td>120</td>
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<tr>
<td>Fixed cost / order</td>
<td>$5,000</td>
<td>$5,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>Optimal order size</td>
<td>1,095</td>
<td>346</td>
<td>110</td>
</tr>
<tr>
<td>Order frequency</td>
<td>11.0 / year</td>
<td>3.5 / year</td>
<td>1.1 / year</td>
</tr>
<tr>
<td>Annual cost</td>
<td>$109,544</td>
<td>$34,642</td>
<td>$10,954</td>
</tr>
</tbody>
</table>

Total cost = $155,140
Aggregation: Order All Products Jointly

\[ S^* = S + s_L + s_M + s_H = 4000 + 1000 + 1000 + 1000 = \$7000 \]

\[ n^* = \sqrt{\frac{D_L h C_L + D_M h C_M + D_H h C_H}{2S^*}} \]

= 9.75

\[ Q_L = \frac{D_L}{n^*} = \frac{12000}{9.75} = 1230 \]

\[ Q_M = \frac{D_M}{n^*} = \frac{1200}{9.75} = 123 \]

\[ Q_H = \frac{D_H}{n^*} = \frac{120}{9.75} = 12.3 \]

Cycle inventory = \( Q/2 \)

Average flow time = \( (Q/2)/(\text{weekly demand}) \)
### Complete Aggregation: Order All Products Jointly

<table>
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<th>Medpro</th>
<th>Heavypro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand per year</td>
<td>12,000</td>
<td>1,200</td>
<td>120</td>
</tr>
<tr>
<td>Order frequency</td>
<td>9.75/year</td>
<td>9.75/year</td>
<td>9.75/year</td>
</tr>
<tr>
<td>Optimal order size</td>
<td>1,230</td>
<td>123</td>
<td>12.3</td>
</tr>
<tr>
<td>Annual holding cost</td>
<td>$61,512</td>
<td>$6,151</td>
<td>$615</td>
</tr>
</tbody>
</table>

**Annual order cost** = $9.75 \times $7,000 = $68,250

**Annual total cost** = $136,528
Lessons from Aggregation

- Aggregation allows firms to lower lot size without increasing cost.
- Complete aggregation is effective if product specific fixed cost is a small fraction of joint fixed cost.
- Tailored aggregation is effective if product specific fixed cost is a large fraction of joint fixed cost.
Economies of Scale to Exploit Quantity Discounts

- All-unit quantity discounts
- Marginal unit quantity discounts
- Why quantity discounts?
  - Coordination in the supply chain
  - Price discrimination to maximize supplier profits
Quantity Discounts

- Lot size based
  - All units
  - Marginal unit
- Volume based

- How should buyer react?
- What are appropriate discounting schemes?
All-Unit Quantity Discounts

- Pricing schedule has specified quantity break points $q_0, q_1, \ldots, q_r$, where $q_0 = 0$
- If an order is placed that is at least as large as $q_i$ but smaller than $q_{i+1}$, then each unit has an average unit cost of $C_i$
- The unit cost generally decreases as the quantity increases, i.e., $C_0 > C_1 > \ldots > C_r$
- The objective for the company (a retailer in our example) is to decide on a lot size that will minimize the sum of material, order, and holding costs
All-Unit Quantity Discount Procedure (different from what is in the textbook)

Step 1: Calculate the EOQ for the lowest price. If it is feasible (i.e., this order quantity is in the range for that price), then stop. This is the optimal lot size. Calculate total cost (TC) for this lot size.

Step 2: If the EOQ is not feasible, calculate the TC for this price and the smallest quantity for that price.

Step 3: Calculate the EOQ for the next lowest price. If it is feasible, stop and calculate the TC for that quantity and price.

Step 4: Compare the TC for Steps 2 and 3. Choose the quantity corresponding to the lowest TC.

Step 5: If the EOQ in Step 3 is not feasible, repeat Steps 2, 3, and 4 until a feasible EOQ is found.
## All-Unit Quantity Discount: Example

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>Unit Price</th>
</tr>
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<tbody>
<tr>
<td>0-5000</td>
<td>$3.00</td>
</tr>
<tr>
<td>5001-10000</td>
<td>$2.96</td>
</tr>
<tr>
<td>Over 10000</td>
<td>$2.92</td>
</tr>
</tbody>
</table>

q0 = 0, q1 = 5000, q2 = 10000  
C0 = $3.00, C1 = $2.96, C2 = $2.92  
D = 120000 units/year, S = $100/lot, h = 0.2
All-Unit Quantity Discount: Example

Step 1: Calculate \( Q_2^* = \sqrt{\frac{2DS}{hC_2}} \)
\[
= \sqrt{\frac{2(120000)(100)}{0.2}(2.92)} = 6410
\]
Not feasible \((6410 < 10001)\)
Calculate \( TC_2 \) using \( C_2 = $2.92 \) and \( q_2 = 10001 \)
\[
TC_2 = \frac{120000}{10001}(100) + \frac{10001}{2}(0.2)(2.92) + (120000)(2.92)
\]
\[
= $354,520
\]

Step 2: Calculate \( Q_1^* = \sqrt{\frac{2DS}{hC_1}} \)
\[
= \sqrt{\frac{2(120000)(100)}{0.2}(2.96)} = 6367
\]
Feasible \((5000 < 6367 \leq 10000) \Rightarrow \text{Stop}\)
\[
TC_1 = \frac{120000}{6367}(100) + \frac{6367}{2}(0.2)(2.96) + (120000)(2.96)
\]
\[
= $358,969
\]
\( TC_2 < TC_1 \Rightarrow \text{The optimal order quantity } Q^* \text{ is } q_2 = 10001 \)
All-Unit Quantity Discounts

- Suppose fixed order cost were reduced to $4
  - Without discount, $Q^*$ would be reduced to 1265 units
  - With discount, optimal lot size would still be 10001 units

- What is the effect of such a discount schedule?
  - Retailers are encouraged to increase the size of their orders
  - Average inventory (cycle inventory) in the supply chain is increased
  - Average flow time is increased
  - Is an all-unit quantity discount an advantage in the supply chain?
Why Quantity Discounts?

- Coordination in the supply chain
  - Commodity products
  - Products with demand curve
    - 2-part tariffs
    - Volume discounts
Coordination for Commodity Products

- $D = 120,000$ bottles/year
- $S_R = $100, $h_R = 0.2$, $C_R = $3$
- $S_S = $250, $h_S = 0.2$, $C_S = $2$

Retailer’s optimal lot size = 6,324 bottles
Retailer cost = $3,795; Supplier cost = $6,009
Supply chain cost = $9,804

<table>
<thead>
<tr>
<th></th>
<th>Supplier</th>
<th>Retailer</th>
<th>Coordinate</th>
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</thead>
<tbody>
<tr>
<td>D</td>
<td>120000</td>
<td>120000</td>
<td>120000</td>
</tr>
<tr>
<td>S</td>
<td>250</td>
<td>100</td>
<td>350</td>
</tr>
<tr>
<td>h</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Q*</td>
<td>12247</td>
<td>6324</td>
<td>9165</td>
</tr>
</tbody>
</table>
Coordination for Commodity Products

- What can the supplier do to decrease supply chain costs?
  - Coordinated lot size: 9,165; Retailer cost = $4,059;
  - Supplier cost = $5,106; Supply chain cost = $9,165

- Effective pricing schemes
  - All-unit quantity discount
    - $3 for lots below 9,165
    - $2.9978 for lots of 9,165 or more
  - Pass some fixed cost to retailer (enough that he raises order size from 6,324 to 9,165)
Quantity Discounts When Firm Has Market Power

- No inventory related costs
- Demand curve
  \[360,000 - 60,000p\]

What are the optimal prices and profits in the following situations?

Production cost: $2

Stages: Manufacturer and Retailer

- The two stages coordinate the pricing decision
  - Price = $4, Profit = $240,000, Demand = 120,000
- The two stages make the pricing decision independently
  - Price = $5, Profit = $180,000, Demand = 60,000
Two-Part Tariffs and Volume Discounts

- Design a two-part tariff that achieves the coordinated solution
- Design a volume discount scheme that achieves the coordinated solution
- Impact of inventory costs
  - Pass on some fixed costs with above pricing

- Two part Tariffs: fixed $180,000 + $2 per bottle
- Volume based discount:
  - if less then 120,000 : $4
  - If equal or greater than 120,000: $3.5
Lessons from Discounting Schemes

- Lot size based discounts increase lot size and cycle inventory in the supply chain
- Lot size based discounts are justified to achieve coordination for commodity products
- Volume based discounts with some fixed cost passed on to retailer are more effective in general
  - Volume based discounts are better over rolling horizon
Trade promotions are price discounts for a limited period of time (also may require specific actions from retailers, such as displays, advertising, etc.)

Key goals for promotions from a manufacturer’s perspective:
- Induce retailers to use price discounts, displays, advertising to increase sales
- Shift inventory from the manufacturer to the retailer and customer
- Defend a brand against competition
- Goals are not always achieved by a trade promotion

What is the impact on the behavior of the retailer and on the performance of the supply chain?

Retailer has two primary options in response to a promotion:
- Pass through some or all of the promotion to customers to spur sales
- Purchase in greater quantity during promotion period to take advantage of temporary price reduction, but pass through very little of savings to customers
Short Term Discounting

$Q^*$: Normal order quantity
$C$: Normal unit cost
$d$: Short term discount
$D$: Annual demand
$h$: Cost of holding $1$ per year
$Q^d$: Short term order quantity

$$Q^d = \frac{dD}{(C - d)h} + \frac{CQ^*}{C - d}$$

- Forward buy $= Q^d - Q^*$
Short Term Discounts: Forward Buying

Normal order size, $Q^* = 6,324$ bottles
Normal cost, $C = $3 per bottle
Discount per tube, $d = $0.15
Annual demand, $D = 120,000$
Holding cost, $h = 0.2$

\[ Q^d = [(0.15)(120000)/(3.00-0.15)(0.2)] + [(3)(6324)/(3.00-0.15)] = 38,236 \text{ bottles} \]

Forward buy = $Q^d - Q^* = 38,236 - 6,324 = 31,912$ bottles
Promotion Pass Through to Consumers

Demand curve at retailer: $300,000 - 60,000\rho$

Normal supplier price, $C^R = $3.00

- Optimal retail price = $4.00
- Customer demand = 60,000

Promotion discount = $0.15

- Optimal retail price = $3.925
- Customer demand = 64,500

*Retailer only passes through half the promotion discount and demand increases by only 7.5%*
Summary of Learning Objectives

- How are the appropriate costs balanced to choose the optimal amount of cycle inventory in the supply chain?
- What are the effects of quantity discounts on lot size and cycle inventory?
- What are appropriate discounting schemes for the supply chain, taking into account cycle inventory?
- What are the effects of trade promotions on lot size and cycle inventory?
- What are managerial levers that can reduce lot size and cycle inventory without increasing costs?
Next Class

- Assignment 1 is uploaded! Due date is next week!
- Safety Inventory