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EFFECT OF MASS ATTACHMENT ON THE FREE VIBRATION OF CRACKED BEAM

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Abstract

The effect of mass attachment on the transverse vibration characteristics of a cracked cantilever beam is investigated. Investigation of the cracked beam has been carried out theoretically. The governing equation for free vibrations of the cracked beam is constructed from the Bernoulli-Euler beam elements. To model the transverse vibration, the crack is represented by a rotational spring. The relative changes of the first three natural frequencies as a function of the location of the attached mass are shown. The crack was located in two different distances from the fixed end of the beam. The results for the changes of the natural frequencies of a cracked beam carrying a point mass are compared with the results of the beam without a crack. In all calculations the beam has a uniform cross-section and the crack was modeled by sawing cuts with a width ratio 0.4. It is well known that when a crack develops in a component, it leads to changes in its natural frequencies. The reducing effects of the cracked beam on the natural frequencies had been more apparent with the mass attached to the beam in different situations. The results can be used to identify cracks in simple beam structures.

INTRODUCTION

The existence of a crack in a beam increases the local flexibility of the beam. The influence of cracks on dynamic characteristics such as changes in natural frequencies, modes of vibration of structures has been the subject of many investigations. Cracked structures have been modeled by various methods. Petroski has modeled the crack by appropriately reducing the section modulus of the beam [1]. This technique has been used by various investigators to study cracked rotors [2, 3]. Another approach has been to model the crack by a local flexibility matrix. The elements of this matrix have been calculated from relations of linear fracture mechanics, the dimensions of which depend upon the degrees of freedom being considered [4]. Dimarogonas and Papadopoulos have computed the flexibility matrix for a transverse surface crack on a shaft [5]. They have modeled the longitudinal and bending vibrations with 2x2-flexibility matrix [6]. For stress analysis purposes, Rice and Levy [7] computed the local flexibility corresponding to tension and bending, including their coupling terms. In the case of pure bending vibrations of beams, this concept reduces to representing the crack by a rotational spring. Chondros and Dimarogonas [8], Dimarogonas and Massouros [9], combined this spring hinge model with fracture mechanics results, and developed a frequency spectral method to identify cracks in various structures. For a known crack position, this method correlated the crack depth to the change in natural frequencies of the first three harmonics of the structure. An extended literature review of these methods can be seen in reference [10].

The effect of the end mass, the rotational moment of inertia, and the distance between the mass center of gravity and the tip a cantilever beam on the modal frequency parameters have been studied in reference [11]. The effects of the support flexibility and the end mass properties on the modal frequencies and the modal shape have been investigated for a large variety of classical and non-classical boundary conditions [12]. The effects of a transverse crack on the modal frequency parameters of stationary shafts carrying elastically mounted end masses has been studied by Dannanh and Farghaly [13].

In this study, the effects of mass attachment on the free vibration of cracked beam carrying a point mass are discussed analytically. Investigation involves in the calculations of the transverse natural frequencies of the beam with a crack and the corresponding uncracked beam. The beam has a uniform cross-section and the calculations are based on the use of rotational spring in order to represent the crack. The relative changes of the first three natural frequencies as a function of the location of the attached mass are shown. The crack was located at two different distances from the fixed end of the cantilever beam. The results of relative natural frequencies can be adopted to predict of location of the crack.

ELASTIC BEHAVIOR OF A CRACKED BEAM CARRYING A POINT MASS

To model the transverse vibration, the crack is represented by a rotational spring of stiffness K_T. The presence of the crack adds a local flexibility to the beam. Dimaragonas and Paipetis [4] calculated the bending spring constant K_T in the vicinity of the cracked section of a beam with orthogonal cross-section of width b and the height h (Figure 1) when a lateral crack of uniform depth a exist, from the crack strain energy function;

$$K_T = \frac{E\,I}{6\pi\left(1-\vartheta^2\right)h\,\Phi(a/h)} \tag{1}$$

where E is the modulus of elasticity of a beam material, I is the moment of inertia of the beam cross-section. The dimensionless local compliance function $\Phi(a/h)$ is computed from the strain energy density function and has the form;

$$\begin{aligned}\Phi(a/h) = {} & 0.6272\,(a/h)^2 - 1.04533\,(a/h)^3 + 4.5948\,(a/h)^4 - 9.973\,(a/h)^5 \\ & + 20.2948\,(a/h)^6 - 33.0351\,(a/h)^7 + 47.1063\,(a/h)^8 - 40.755\,(a/h)^9 \\ & + 19.6\,(a/h)^{10}\end{aligned} \tag{2}$$

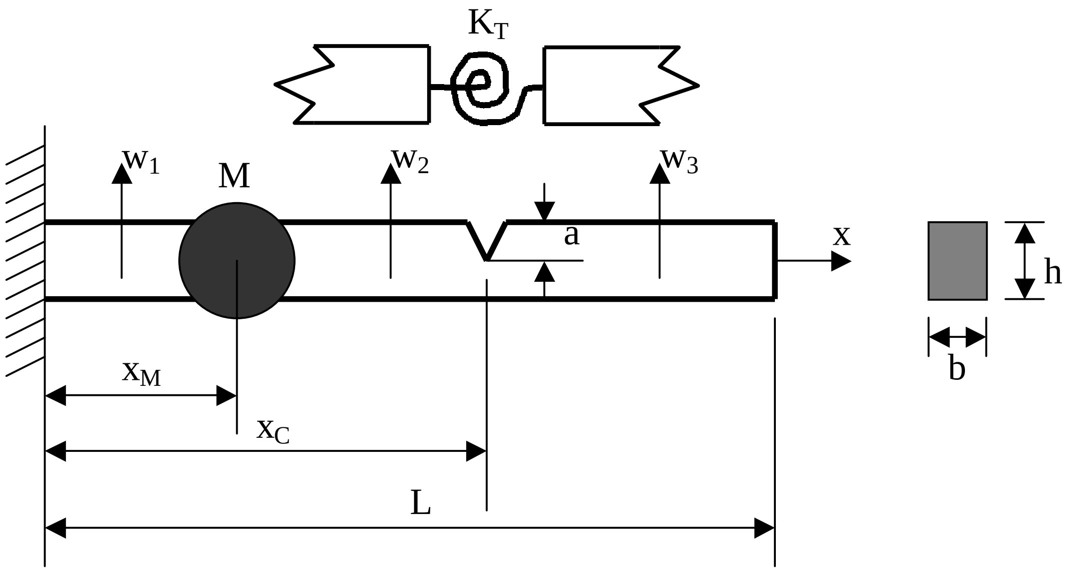

Figure 1. Cracked cantilever beam with a point mass

The bending vibrations of a uniform Bernoulli-Euler beam are governed by the partial differential equation**;**

$$E\,I\,\frac{\partial^4 w(x,t)}{\partial x^4} + \rho\,A\,\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{3}$$

where w(x,t) denotes the lateral displacement at point x at time t. A is the cross-sectional area and ρ is the mass density.

The bending displacements in the regions to the left and right of the in-span attachment of the mass will be denoted as $w_1(x,t)$ and $w_2(x,t)$ where both are subject to the differential equation (3). The corresponding matching conditions at the mass location $(\beta_M = x_M/L)$ are;

$$w_1(\beta_M, t) = w_2(\beta_M, t), \quad w_1'(\beta_M, t) = w_2'(\beta_M, t), \quad w_1''(\beta_M, t) = w_2''(\beta_M, t)$$
$$E I \left(w_1'''(\beta_M, t) - w_2'''(\beta_M, t)\right) = M \ddot{w}_2(\beta_M, t) \tag{4}$$

The crack is assumed to be open and to have uniform depth. The beam can be conveniently divided into two segments, one on either side of the spring representing the crack. The bending displacements in the regions to the left and right of the crack will be denoted as $w_2(x,t)$ and $w_3(x,t)$. The continuity of displacement, moment and shear forces at the crack location $(\beta_C = x_C/L)$ and jump condition in the slope can be written in the following form:

$$w_2(\beta_C, t) = w_3(\beta_C, t), \quad w_2''(\beta_C, t) = w_3''(\beta_C, t), \quad w_2'''(\beta_C, t) = w_3'''(\beta_C, t)$$
$$w_3'(\beta_C, t) = w_2'(\beta_C, t) + E I\, w_3''(\beta_C, t)/K_T \tag{5}$$

The boundary conditions at the fixed and free ends, respectively, are**;**

$$w_1(0, t) = 0, \quad w_1'(0, t) = 0, \quad w_3''(1, t) = 0, \quad w_3'''(1, t) = 0 \tag{6}$$

If harmonic solutions are assumed for three regions

$$w_i(x, t) = Y_i(x) \cos(\omega t - \varphi) \qquad i = 1, 2, 3 \tag{7}$$

The harmonic vibrations on the three parts of the beam are;

$$\begin{aligned} Y_1(\beta) &= A_1 \cos(\lambda\beta) + A_2 \sin(\lambda\beta) + A_3 \cosh(\lambda\beta) + A_4 \sinh(\lambda\beta) \\ Y_2(\beta) &= A_5 \cos(\lambda\beta) + A_6 \sin(\lambda\beta) + A_7 \cosh(\lambda\beta) + A_8 \sinh(\lambda\beta) \\ Y_3(\beta) &= A_9 \cos(\lambda\beta) + A_{10} \sin(\lambda\beta) + A_{11} \cosh(\lambda\beta) + A_{12} \sinh(\lambda\beta) \end{aligned} \tag{8}$$

where A_i i = 1,…..,12, are arbitrary integration constant to be evaluated from the boundary and matching conditions for w_1, w_2 and w_3. In order to obtain non-vanishing solutions for A_1,….A_{12} , the corresponding determinant of coefficients has to be equated to zero. The determinant equation in which the following non-dimensional parameter is introduced:

$$\lambda^4 = \omega^2 \rho A L^4 / E I, \quad \beta = x/L, \quad \gamma_M = M/(\rho A L), \quad \gamma_C = E I / K_T L \tag{9}$$

Here, ω is the natural angular frequency, x is the coordinate along the beam, with the origin at the clamp end, L is the length of the beam. γ_M and γ_C represent the mass ratio and dimensionless local flexibility coefficient, respectively.

The roots of the determinant, which are obtained numerically, provide not only the dimensionless frequency parameters λ, but also the natural frequencies of the system in Figure 1, by considering the relation (9).

NUMERICAL APPLICATION

The effect of the attachment of the point mass on the first three natural frequencies of the cantilever beam with a crack is given in form figures. Calculation in this study was carried out via the following beam data: length 1m, height 0.01 m, width 0.01 m, Young's modulus $E = 2.1 10^{11}$ Pa, Poisson ratio $\vartheta = 0.3$, density $\rho = 7860\ kg/m^3$. Two different crack location parameters $\beta_C = 0.5, 0.7$ together with a constant crack depth ratio $a/h = 0.4$ are considered.

In Figure 2-4, the variation of the first three relative natural frequencies (normalized to the corresponding frequency ω_{no} of a beam without a crack and a point mass = ω/ω_{no}) are depicted as a function of the non-dimensional location parameter of the attached mass in the range of zero to one. The mass parameter is kept constant as $\gamma_M = 0.3$. In the absence of the crack i.e. $(K_T \to \infty)$ results are obtained for the uncracked beam.

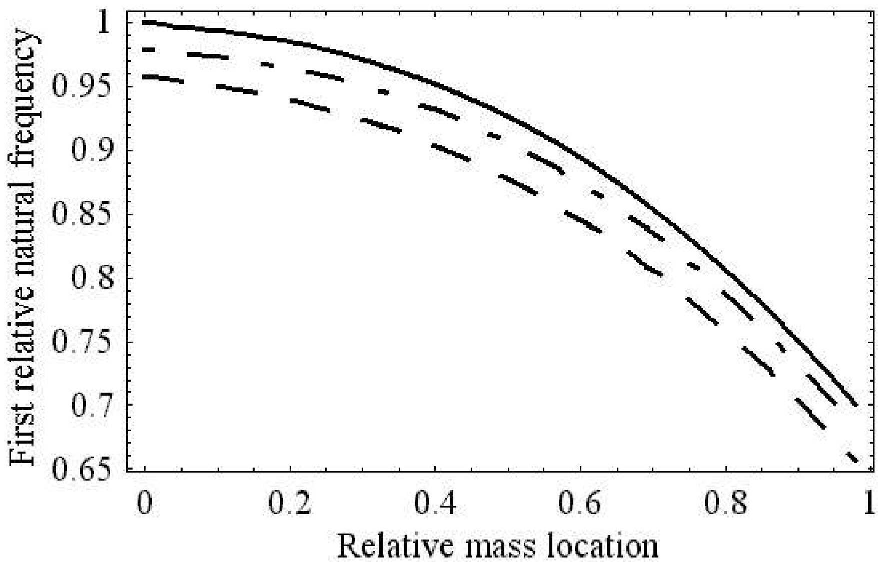

Figure 2. The relative changes of the first natural frequency as a function of the mass location for a cantilever beam. ——, uncracked; — — ,β_C =0.5; — - —, β_C =0.7 (a/h = 0.4, γ_M = 0.3).

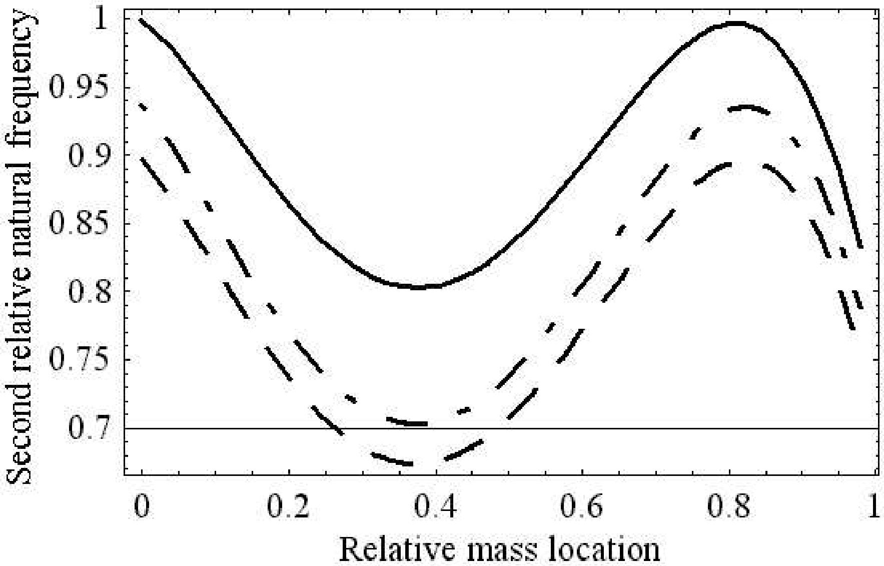

Figure 3. The relative changes of the second natural frequency as a function of the mass location for a cantilever beam ——, uncracked; – –, β_C =0.5; — – —, β_C =0.7 (a/h = 0.4, γ_M = 0.3).

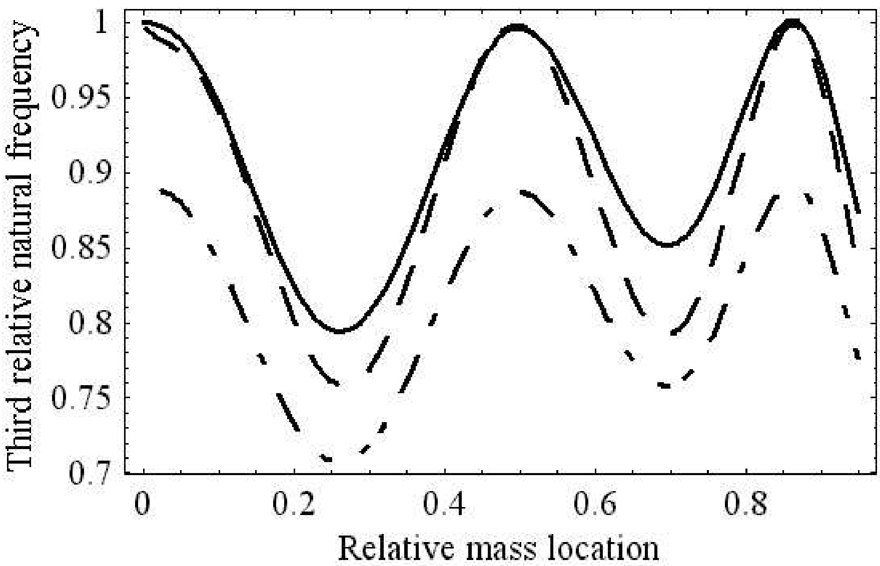

Figure 4. The relative changes of the third natural frequency as a function of the mass location for a cantilever beam ——, uncracked; – –, β_C =0.5; — – —, β_C =0.7 (a/h = 0.4, γ_M = 0.3).

As can be seen in the figures 2, 3 and 4, mass attachments to the both cracked and uncracked beams have led to declines in all three natural frequencies. It is known that the crack reduces the natural frequencies. The reducing impact of the current crack on the natural frequencies with the mass attached to the beam in different situations becomes more evident.

In figure 3, the variation of the first natural frequency is seen. As the location of the mass attachment moves from the fixed end to the free end, the first natural frequency for both the cracked and uncracked beam continuously falls. If the location of the crack is close to the fixed side (β_C=0.5), this reduction is higher than the case of its location farther (β_C=0.7) from the fixed side. If the crack will be determined via the observation the first natural frequency, it is appropriate to attach the mass to the free end, since it will lead to the highest change.

In figure 4, the variation of the second natural frequency is given. As the mass moves closer to the free end, the second natural frequency initially declines, later it displays an increasing trend and at 0.8, it becomes indifferent to the mass. The change in the frequency at the creak's β_C=0.5 position is higher than the change at its position of β_C=0.7. In terms of providing the highest change in the second natural frequency, the most effective point is observed at β_M = 0.4, where the mass is defined as dimensionless situation parameter.

In figure 5, the variation of the third natural frequency depending on the situation of the mass attachment is given. Until the mass reaches the end, the natural frequency first declines, then increases and at β_M = 0.5 it becomes unresponsive. After this point, as the mass moves to the free end, first an increase than again a decrease are observed, while it is indifferent around β_M = 0.85. If the crack is at β_C=0.5 position, at β_M = 0.5 and β_M = 0.85 points the third natural frequency reaches its levels at beam's uncracked position without mass attachment. It will be impossible to determine the position of the crack with the analysis at these points. Small differences are recorded at the other situations of the mass. If information is required about the situation of the crack depending on the variations in the third natural frequency, it can be said that the crack position at β_C=0.5 will not lead to a significant change with the mass attachment. If the crack is at β_C=0.7 position, its reducing impact on the natural frequency is seen for all locations of the mass. It is seen in figure 5 that mass attachment at around β_M = 0.25 will lead to the highest variation in the third natural frequency.

CONCLUSIONS

The relative changes of the first three natural frequencies as a function of the location of the attached mass are investigated. The crack was located at two different distances from the fixed end of the beam. The results for the changes of the natural frequencies of a cracked beam carrying a point mass are compared with the results of the beam without a crack. In all calculations the beam has a uniform cross-section and the crack was modeled by sawing cuts with a width ratio 0.4.

In some cases it is observed that the mass attachment has no impact on the frequencies for the beams with cracks. These unresponsive cases alter according to the observed frequency. The distance of the crack from the fixed end also creates different effects depending on the frequency. For the cantilever beam, the second natural frequency is mostly affected with the crack position at $\beta_C = 0.5$, while in some situations of the mass, the third natural frequency reaches its without mass and crack levels. The mass is most effective at the free end of the beam for the first natural frequency, at $\beta_M = 0.4$ point from the fixed end for the second natural frequency and at $\beta_M = 0.25$ point for the third natural frequency.

It is observed that with the mass attachment in various situations, cracks in structures have a clearer decreasing impact on the natural frequencies. The results can be used to identify cracks in simple beam structures.

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