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ACOUSTIC ATTENUATION IN FULLY-FILLED PERFORATED DISSIPATIVE MUFFLERS WITH EXTENDED INLET/OUTLET

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Abstract
Exhaust noise of IC engines is the main component of noise pollution of the urban environment. To attenuate the exhaust noise, expansion chambers with different configuration has become an important area of research and development. The present study considers acoustic characteristics of circular fully-filled perforated dissipative mufflers with extended inlet/outlet. In addition to the finite element method, an experimental work is considered. Fiber material with different radii is presented and discussed. Perforation ratios of the pipes are also discussed in this study. All parameters validated with experimental work with a practical designed muffler for obtaining the transmission loss. Comparison with the results obtained via experimental measurements justifies the approaches used here.

INTRODUCTION

Expansion chambers with extended pipes exhibit a desirable acoustic attenuation performance as a combination of usually broadband domes of a simple expansion chamber and the resonant peaks of a quarter-wave resonator. Various selections of lengths of extended ducts cause a great improvement on the acoustic behaviour of silencers over a wide frequency range. Furthermore, recent works in fiber properties combined with their broadband acoustic dissipation characteristics and perforated ducts make such mufflers potentially desirable for their acoustic characteristics. The acoustic attenuation of the exhaust muffler is important for automotive engineers. Exhaust muffler designers often use fiber materials and perforated pipes in dissipative mufflers in order to improve the attenuation over a wide frequency range.

Different methods for modelling dissipative mufflers have been studied previously by the researchers. Xu *et al.* [1], by using a two-dimensional analytical approach, examined the effect of fiber thickness, chamber diameter and material properties on the acoustic performance of dissipative silencers. An analytical approach was proposed based on the solution of eigenequation for a circular dissipative expansion chamber. The acoustic pressure and particle velocity across the silencer discontinuities were matched by imposing the continuities of the velocity/pressure integral over discrete zones at the expansion/contraction. Kirby [2], presented a method that two-dimensional finite element eigenvalue calculation is combined with a point collocation matching scheme in the inlet/outlet ducts for mufflers with mean flow and perforated pipe. This method showed a good agreement with experimental measurements. Selamet *et al.* [3], combined the behaviour of extended ducts and the effects of dissipation of absorbing materials. A hybrid silencer was studied both by methods of boundary element method and multi-dimensional approach to see the effects of variations in the structure of the muffler. Finite element method was commonly discussed for modelling mufflers by means of taking three-dimensional effects into account. Munjal [4], considered methods for solving Galerkin formulation and also Mehdizadeh *et al.* [5], implemented a three dimensional finite element method to predict the transmission loss of a muffler for a wide frequency range. In those studies, a hybrid muffler with perforated pipes and absorbing materials was modelled for obtaining transmission loss (TL) characteristics by using finite element methods. The results obtained via numerical methods were compared with experiments which they had good agreements. Several experimental works were implemented for different methods. Calculating four-pole parameters is the most common way of predicting transmission loss. Munjal *et al.* [6], who proposed a two-source method for measuring the four-pole parameters of an acoustic element or combination of elements.

In this study, the aim is to predict the acoustic performance of fully-filled perforated dissipative mufflers with extended inlet/outlet. In order to control quarter wave resonator frequencies, absorbing material positioned in the central region of the muffler. Finite element approach is applied to predict four pole parameters for various different combinations of mufflers to obtain transmission loss characteristics. Experimental studies are done to provide results of the finite element method approach.

THEORY

The system to be dealt with in the present study is shown diagrammatically in Fig. 1. Let it be assumed that the radius of the inlet and outlet pipes is r, whereas the length and radius of the chamber is L and R, respectively. Also, the thickness of the fiber material is t. The linear wave equation for a perfect gas with no damping,

$$\frac{\partial^2 P}{\partial t^2} = c^2 \nabla^2 P \tag{1}$$

where P is the sound pressure and c is the speed of sound. If equation (1) is solved for corresponding boundary and initial conditions in the time domain, it gives P as a function of time and space.

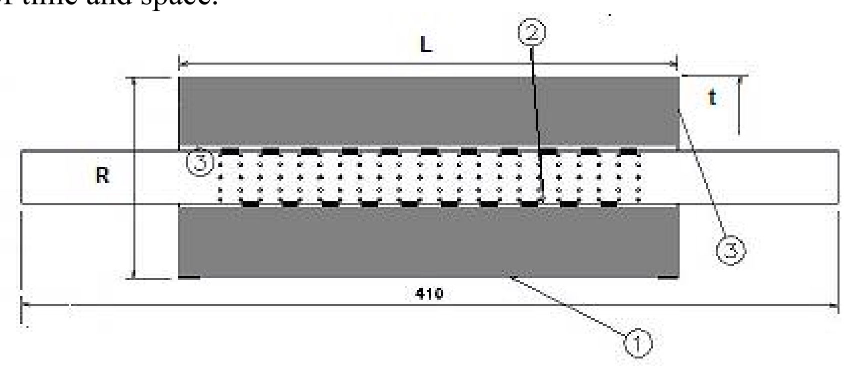


Figure 1 – Muffler with perforated pipe and porous material.

The sound pressure assumed for time-harmonic solution is

$$P = pe^{i\omega t}. \tag{2}$$

Then the linear wave equation becomes Helmholtz's equation as

$$\nabla^2 p = -k^2 p, \tag{3}$$

where p is in the frequency domain. For mufflers with porous materials, the governing equations become

$$\nabla^2 p_a + k_a{}^2 p_a = 0 \text{ in } \Omega_a \text{ and } \nabla^2 p_b + k_b{}^2 p_b = 0 \text{ in } \Omega_b \tag{4}$$

where Ω_a and Ω_a are the domains of air and bulk porous material respectively. The boundary conditions are

$$p = 1, \text{ at the inlet, } \quad \frac{\partial p}{\partial n} = 0, \text{ on rigid walls, } \frac{\partial p}{\partial n} = ikp, \text{ at the outlet.} \tag{5}$$

One-dimensional plane wave propagation is assumed at the inlet and the outlet pipes. Hence, at the outlet, the anechoic boundary condition identified in equation (5) is a rather simple form of a Robin boundary condition. The normal velocity is continuous and the normal pressure gradient is proportional to density ratio at the interface between air and porous material, thus represented as

$$u_a \cdot n_a = -u_b \cdot n_b, \frac{\partial p_b}{\partial n} = -\frac{\rho_b}{\rho_a}\frac{\partial p_a}{\partial n}. \tag{6}$$

On the perforated pipe, the basic assumptions for modelling a perforated plate in an acoustic field are a continuous normal velocity and a discontinuous pressure through the pipe. The simple relation proposed by Sullivan and Crocker [7] is considered sufficiently accurate for usage in this paper. The dimensionless transfer impedance Z_t of a perforated plate can be approximated as follows

$$Z_t = \frac{1}{\rho_0 c \sigma}(2.4 + i0.02 f), \tag{7}$$

where σ is the ratio of the open area to the total area of the pipe. The pressure jump, Δp, normal particle velocity, u_n and normal sound pressure gradient relation are calculated by following equations, respectively.

$$\Delta p = \rho_0 c Z_t u_n, \quad \frac{\partial p}{\partial n} = -i\rho_0 \omega u_n. \tag{8}$$

Note that equation (7) is valid only for a perforated pipe surrounded by air, so a narrow air gap between the absorption material and the perforated pipe is assumed. At this point, following continuous space of complex functions must be introduced as

$$Z = \left\{ \upsilon = \upsilon_{\mathrm{Re}} + i\upsilon_{\mathrm{Im}} : \upsilon_{\mathrm{Re}} \in H^1(\Omega), \upsilon_{\mathrm{Im}} \in H^1(\Omega) \right\} \tag{9}$$

where H^1 is the Hilbert space. The variatonal formulation of the problem is to find $p \in Z$ such that

$$\begin{aligned} &\int_{\Omega_a} (\nabla\upsilon \cdot \nabla p_a - k_a^2 \upsilon p_a) d\Omega + \int_{\Omega_b} \frac{\rho_a}{\rho_b} (\nabla\upsilon \cdot \nabla p_b - k_b^2 \upsilon p_b) d\Omega \\ &+ \int_{S_{p1}} i\frac{k_a}{Z_t} \upsilon (p_{p1} - p_{p2}) dS + \int_{S_{p2}} i\frac{k_a}{Z_t} \upsilon (p_{p2} - p_{p1}) dS \\ &- \int_{S_0} ik_a \upsilon p_0 dS = 0 \end{aligned} \tag{10}$$

where S_0 is the cross-sectional area of the output pipe and S_{p1} and S_{p2} are surfaces of perforated plate. In equation (10), volume integrals (the first two terms) governing air and porous material, surface integrals (the next two terms) governing pressure jump through perforated pipe and linked the pressure in both sides and the last integral surface term is the Robin boundary condition for modelling anechoic termination at the outlet. Common Galerkin formulation method is utilized for solving the equation. Three-dimensional domain, Ω is divided into K tetrahedral elements.

$$\overline{\Omega} = \cup_{j=1}^{K} \overline{\hat{\Omega}} \tag{11}$$

The discreet approximation space, $H_h^1 \subset H^1$, is defined as

$$H_h^1 = \left\{ \upsilon = \upsilon_{\mathrm{Re}} + i\upsilon_{\mathrm{Im}} : \upsilon_{\mathrm{Re}} \in P_2(\hat{\Omega}_j), \upsilon_{\mathrm{Im}} \in P_2(\hat{\Omega}_j) \right\} \tag{12}$$

where P_2 are polynomials of degree two defined on each tetrahedral elements, j. The global basis functions,

$$\phi_n(X_m) = \delta_{nm}, \quad 1 \le n,\ m \le N, \tag{13}$$

where N is the total number of global nodes in the model. X is the coordinate of each nodes. By using global basis functions, υ, can be

$$\upsilon(X) \approx \upsilon_h(X) = \sum_{m=1}^{N} \upsilon_m \phi_m(X) \tag{14}$$

where $\upsilon_m = \upsilon(X_m)$. Using equation (14), expand all functions to variational formulation, the discretized equation of linear systems of algebraic equations such that

$$\mathbf{Ap=f} \tag{15}$$

where the coefficient matrix **A** is a sparse symmetric matrix, **p** is the sound pressure amplitude vector of nodal values and **f** is forcing function vector of nodal values. In this equation **f** is only a non-zero value at the inlet pipe according to Dirichlet boundary conditions. All these values are complex numbers at all and can be written

$$(\mathbf{A_{Re}}+i\mathbf{A_{Im}})(\mathbf{p_{Re}}+i\mathbf{p_{Im}})=\mathbf{f_{Re}}+i\mathbf{f_{Im}} \tag{16}$$

Equation (16), can be written in matrix form as

$$\begin{bmatrix} \mathbf{A_{Re}} & \mathbf{A_{Im}} \\ \mathbf{A_{Im}} & \mathbf{-A_{Re}} \end{bmatrix} \begin{bmatrix} \mathbf{p_{Re}} \\ \mathbf{-p_{Im}} \end{bmatrix} = \begin{bmatrix} \mathbf{f_{Re}} \\ \mathbf{f_{Im}} \end{bmatrix} \tag{17}$$

which above equation is solved by several iterative solvers developed before. The components of the coefficient matrix **A**, A_{mn} are

$$\begin{aligned} A_{mn} &= \int_{\Omega_a} (\nabla\phi_m \cdot \nabla\phi_n - k_a^2 \phi_m \phi_n) d\Omega + \int_{\Omega_b} \frac{\rho_a}{\rho_b} (\nabla\phi_m \cdot \nabla\phi_n - k_b^2 \phi_m \phi_n) d\Omega \\ &+ \int_{S_{p1}} i \frac{k_a}{Z_t} \phi_m (\phi_n - \phi_n) dS + \int_{S_{p2}} i \frac{k_a}{Z_t} \phi_m (\phi_n - \phi_n) dS \\ &- \int_{S_0} i k_a \phi_m \phi_n dS \end{aligned} \tag{18}$$

When applying Equation (18) to each components to evaluate the global matrix **A**, all local coordinates (x,y,z) transformed into global coordinate system (ξ_1,ξ_2,ξ_3). For tetrahedral elements, the coordinates of reference elements are functions of the elemental basis functions, $h_i(\xi_l)$. The components of the elemental matrix $\hat{\mathbf{A}}$, $\hat{A}_{ij}$ for an internal element, $\hat{\Omega}$ can be written as

$$\begin{aligned} \hat{A}_{ij} &= \int_{\hat{\Omega}} \frac{1}{|J|} \Bigg[\left(\sum_{l=1}^{3} J \frac{\partial h_i}{\partial \xi_l} \frac{\partial \xi_l}{\partial x} \right) \left(\sum_{i=1}^{3} J \frac{\partial h_j}{\partial \xi_l} \frac{\partial \xi_l}{\partial x} \right) \\ &+ \left(\sum_{i=1}^{3} J \frac{\partial h_i}{\partial \xi_l} \frac{\partial \xi_l}{\partial y} \right) \left(\sum_{i=1}^{3} J \frac{\partial h_j}{\partial \xi_l} \frac{\partial \xi_l}{\partial y} \right) \\ &+ \left(\sum_{i=1}^{3} J \frac{\partial h_i}{\partial \xi_l} \frac{\partial \xi_l}{\partial z} \right) \left(\sum_{i=1}^{3} J \frac{\partial h_j}{\partial \xi_l} \frac{\partial \xi_l}{\partial z} \right) \Bigg] d\hat{\Omega} - k^2 \int_{\hat{\Omega}} |J| h_i h_j d\hat{\Omega} \end{aligned} \tag{19}$$

which $1 \leq i, j \leq 10$ interval. Elemental integration is numerically performed by Gaussian quadrature which is defined on the reference element.

In this study, the finite element method approach is done by MSc.Actran that is a commercial package program which has a wide library of acoustic elements and materials. Porous media and perforated pipe can be modelled in the program.

EXPERIMENTAL SETUP

The common procedure for measuring the transmission loss (TL) of a muffler is to determine the incident power by decomposition theory and the transmitted power by the plane wave approximation assuming an anechoic termination at the outlet. Unfortunately, it is difficult to determine a fully anechoic termination. Furthermore, there are two alternative measurement techniques which are considered not require an anechoic termination: the two load method and the two source method. In this paper two-load method is applied. An acoustical element, like a muffler, can also be modelled to obtain four-pole parameters. Assuming plane wave propagation at the inlet and outlet, the four-pole method is to relate the pressure and velocity (particle, volume or mass) at the inlet to that at the outlet. Using the four-pole parameters, the transmission loss of a muffler can also be readily calculated. The two-load method, which is based on transfer matrix method, can be modelled by its four-pole parameters, as show in Figure 2.

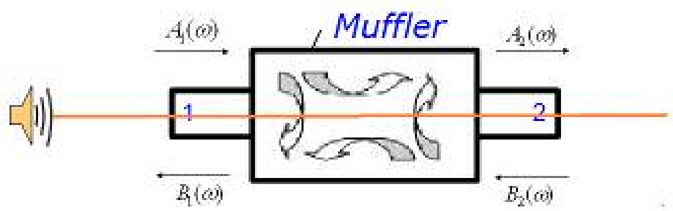


Figure 2 – Four-pole parameters for experimental setup.

The transfer matrix is shown below

$$\begin{Bmatrix} A_1(\omega) \\ B_1(\omega) \end{Bmatrix} = \begin{pmatrix} \alpha(\omega) & \beta(\omega) \\ \gamma(\omega) & \delta(\omega) \end{pmatrix} \begin{Bmatrix} A_2(\omega) \\ B_2(\omega) \end{Bmatrix} \tag{20}$$

where $\alpha(\omega)$, $\beta(\omega)$, $\gamma(\omega)$, $\delta(\omega)$ are the four-pole parameters; $A_1(\omega)$, $B_1(\omega)$ and $A_2(\omega)$, $B_2(\omega)$ are the pressure and velocities at the inlet and outlet, respectively. Application is based on the transfer matrix method, which muffler must be placed like in the Figure 3. Also, by changing the end condition (acoustic impedance) with two separate measurements. Generally, two loads can be two different length pipes, a single pipe with and without absorbing material or even two different mufflers. In this paper, two loads method were achieved by a pipe with and without absorbing material. Thus, transmission loss is calculated by

$$TL(\omega) = 20Log_{10}\left|\alpha(\omega)\right| \tag{21}$$

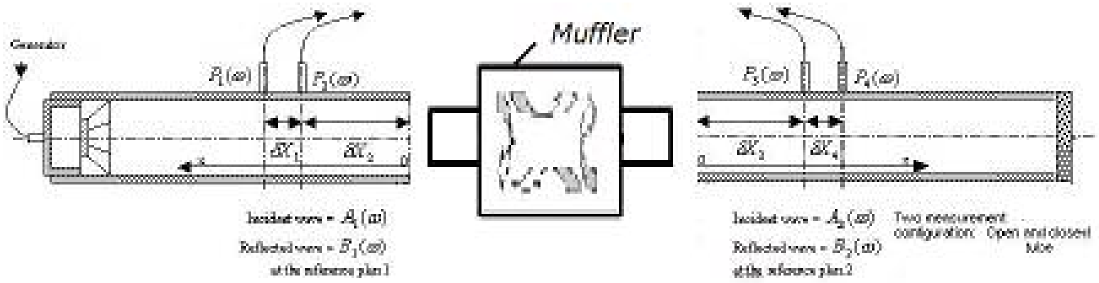
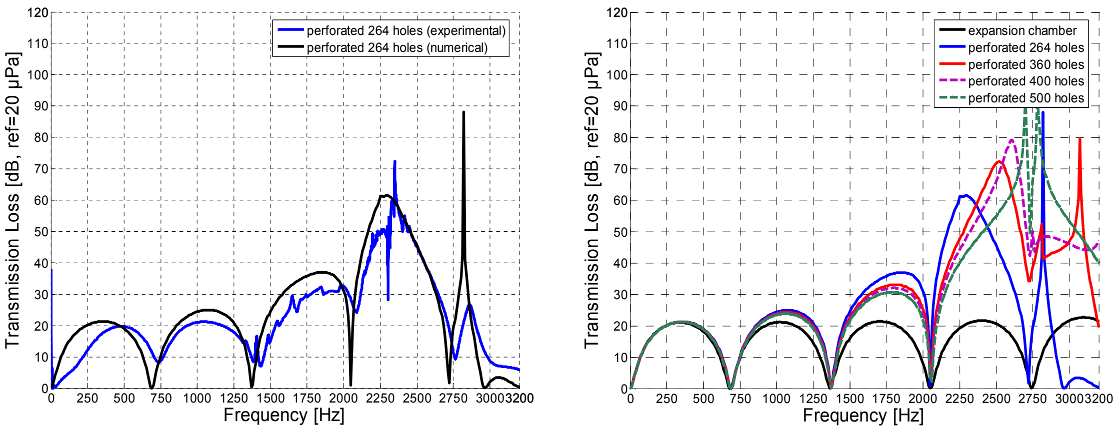


Figure 3 – Impedance Tube for experiments to calculate transmission loss.

RESULTS

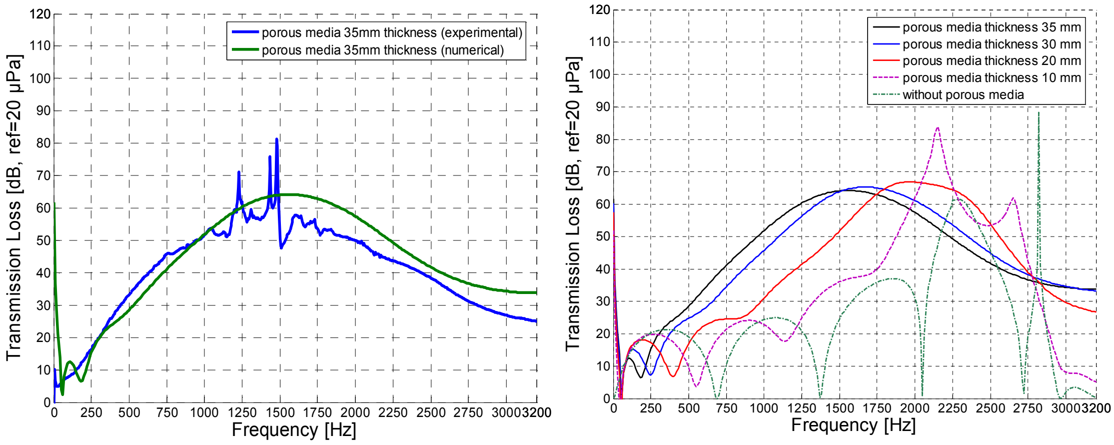
Experimental muffler has dimensions of a simple expansion chamber with perforated pipe and acoustic fiber lining of rock wool material. The radius of the inlet and outlet pipes is r = 2.7 cm, whereas the length and radius of the chamber is L = 25 cm and R = 10 cm, respectively. Also, the thickness of the rock wool is t = 3.5 cm. The rock wool flow resistivity equals 13813 MKS rayls/m. The frequency range of interest is between 1-3200 Hz both in numerical and experimental evaluations. The element size for the finite element domain was chosen to provide a minimum 6 elements per wavelength. It has 101000 elements for cavity domain and 26000 elements for porous material domain. MSc.Actran evaluates this type of a muffler within 17 minutes.



(a) *(b)*

Figure 4 –Transmission loss of mufflers with perforated pipe, (a) numerical and experimental results, (b) different perforation ratios.

Transmission losses of mufflers with and without porous media are given in the Figure 4 and 5, respectively. To provide the FEM results, experimental calculation is given with these figures. It can be seen from figures that the experimental and numerical calculations shows good agreement both for perforated mufflers and perforated mufflers with porous material. Some mismatch within the transmission loss values around 1.6 kHz frequencies in Figure 5 can be negligible because of mathematical modelling. The mathematical model assumes no absorption in air and a perfect reflection on the walls which is not accurate. Another reason is that, the porous material and the perforated plates are approximated by complex acoustic impedance evaluated using rather simple empirical relations. This may explain the errors in the results.



(a) *(b)*

Figure 5 –Transmission loss of mufflers with perforated plate and porous media, (a) numerical and experimental results, (b) different perforation ratios.

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