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# ACOUSTIC ATTENUATION OF A CIRCULAR EXPANSION CHAMBER INCLUDING FIBER MATERIAL AND EXTENDED INLET/OUTLET TUBES 

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#### Abstract

Exhaust noise of IC engines is the main component of noise pollution of the urban environment. With the ever increasing population density of vehicles on the road, this has become an important area of research and development. Most of the advances in the theory of acoustic filters and exhaust mufflers have come about in the last four decades. Due to their desirable broadband noise attenuation characteristics expansion chambers with different configurations are widely used in the exhaust systems. The present study addresses essentially to the investigation of the acoustic attenuation performance of a circular expansion chamber including fiber material and extended inlet/outlet tubes, in which a theoretical analysis has been made in order to determine the transmission loss of the system as the acoustic performance parameter. First, a two dimensional analytical solution for the transmission loss is established via boundary value formulation for the concentric configuration of the circular expansion chamber having fiber material and extended tubes. It is important to note that the solutions obtained are expressed in terms of parameters characterizing the physical properties of the system under consideration. In order to compare the solutions, transmission loss of the system is also computed by using the finite element method model. All equations are then numerically solved for various combinations of physical parameters and the results are represented in figures. The comparison of the numerical results obtained via analytical formulations justifies the finite element method approach used here.


## INTRODUCTION

Expansion chambers with extended inlet/outlet tubes exhibit a desirable acoustic attenuation performance as a combination of usually broadband domes of a simple expansion chamber and the resonant peaks of a quarter-wave resonator. Furthermore, recent improvements in
fiber properties combined with their broadband acoustic dissipation characteristics make such materials potentially desirable for implementation in silencers. In reference [1], using a twodimensional analytical approach, the study examines the effect of fiber thickness, chamber diameter, and material properties on the acoustic performance of dissipative silencers. An analytical approach is proposed based on the solution of eigenequation for a circular dissipative expansion chamber. The acoustic pressure and particle velocity across the silencer discontinuities are matched by imposing the continuities of the velocity/pressure integral over discrete zones at the expansion/contraction. In reference [2], the acoustic attenuation performance of circular expansion chambers with extended inlet/outlet is investigated. Three approaches are employed to determine the transmission loss: first, a two-dimensional, axisymmetric analytical solution for the concentric configuration; second, a threedimensional computational solution based on the substructure boundary element transfer impedance matrix technique; and finally, experiments on an extended impedance tube set-up with expansion chambers fabricated with fixed inlet, outlet, and chamber diameters, and varying lengths for the extended ducts and the chamber, and varying offset locations of the inlet and outlet. The present study addresses essentially to the investigation of the acoustic attenuation performance of a circular expansion chamber including fiber material and extended inlet/outlet tubes, in which a theoretical analysis has been made in order to determine the transmission loss of the system as the acoustic performance parameter. A two dimensional analytical solution for the transmission loss is established via boundary value formulation for the concentric configuration of the circular expansion chamber having fiber material and extended tubes.

## THEORY

Consider a cylindrical expansion chamber, with extended inlet/outlet tubes, having length L and radius $r_{4}$, with sound-absorbing material placed between radius $r_{3}$ and $r_{4}$, as shown in figure 1. The inlet and outlet radiuses are $r_{1}$ and $r_{2}$, respectively. The absorbing material is assumed to be homogeneous and isotropic, characterized by the complex speed of sound $\widetilde{c}$ and density $\tilde{\rho}$. In the inlet pipe, the solution of the Helmholtz equation

$$
\begin{equation*}
\frac{\partial^{2} P}{\partial r^{2}}+\frac{1}{r} \frac{\partial P}{\partial r}+\frac{\partial^{2} P}{\partial x^{2}}+k_{0}^{2} P=0 \tag{1}
\end{equation*}
$$

can be written as

$$
\begin{equation*}
P_{A}(r, x)=\sum_{0}^{\infty}\left(A_{n}^{+} e^{-j k_{x, A, n} x}+A_{n}^{-} e^{j k_{x, A, n} x}\right) \psi_{A, n}(r) \tag{2}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{A}}$ is the acoustic pressure in domain $\mathrm{A}, k_{0}$ is the wavenumber in air with $\mathrm{c}_{0}$ being the speed of sound. $k_{\mathrm{x}, \mathrm{A}, \mathrm{n}}$ is the wavenumber in x direction. The eigenfunctions for the circular duct,

$$
\begin{equation*}
\psi_{A, n}(r)=J_{0}\left(\frac{\alpha_{n}}{r_{1}} r\right) \tag{3}
\end{equation*}
$$

where $\mathrm{J}_{0}$ is the zeroth order Bessel function of the first kind. $\frac{\alpha_{n}}{r_{1}}$ is the radial wavenumber which satisfies the rigid wall boundary condition of

$$
\begin{equation*}
\left.\frac{\partial \psi_{A, n}(r)}{\partial r}\right|_{r=r_{1}}=0 \tag{4}
\end{equation*}
$$

The particle velocity in x direction may then be written,

$$
\begin{equation*}
\rho_{0} \frac{\partial u_{x}}{\partial t}=-\frac{\partial P}{\partial x}, \quad \mathrm{u}_{\mathrm{x}, \mathrm{~A}}(\mathrm{r}, \mathrm{x})=\frac{1}{\rho_{0} \omega} \sum_{n=0}^{\infty} k_{x, A, n}\left(A_{n}^{+} e^{-j k_{x, A, n} x}-A_{n}^{-} e^{j k_{x, A, n} x}\right) \psi_{A, n}(r) \tag{5}
\end{equation*}
$$

where $\rho_{0}$ is the density of air and $\omega$ the angular velocity.


Figure 1 - Circular expansion chamber including fiber material and extended inlet/outlet tubes.
In the outlet pipe, the acoustic pressure and axial velocity are similar to those in the inlet pipe, and can be expressed as

$$
\begin{align*}
P_{E}(r, x)= & \sum_{n=0}^{\infty}\left(E_{n}^{+} e^{-j k_{x, E, n}\left(x-l_{C}\right)}+E_{n}^{-} e^{j k_{x, E, n}\left(x-l_{C}\right)}\right) \psi_{E, n}(r), \quad \psi_{E, n}(r)=J_{0}\left(\frac{\alpha_{n}}{r_{2}} r\right), \\
& \mathrm{u}_{\mathrm{x}, \mathrm{E}}(\mathrm{r}, \mathrm{x})=\frac{1}{\rho_{0} \omega} \sum_{n=0}^{\infty} k_{x, E, n}\left(E_{n}^{+} e^{-j k_{x, E, n}\left(x-l_{C}\right)}-E_{n}^{-} e^{j k_{x, E, n}\left(x-l_{C}\right)}\right) \psi_{E, n}(r) . \tag{6}
\end{align*}
$$

For concentric annular rigid duct, in domain $B$ the solution to the Helmholtz equation can be written

$$
\begin{gather*}
P_{B}(r, x)=\sum_{n=0}^{\infty}\left(B_{n}^{+} e^{-j k_{x, B, n} x}+B_{n}^{-} e^{j k_{x, B, n} x}\right) \psi_{B, n}(r),  \tag{7}\\
\mathrm{u}_{x, B}(\mathrm{r}, \mathrm{x})=\frac{1}{\rho_{0} \omega} \sum_{n=0}^{\infty} k_{x, B, n}\left(B_{n}^{+} e^{-j k_{x, B, n} x}-B_{n}^{-} e^{j k_{x, B, n} x}\right) \psi_{B, n}(r) \tag{8}
\end{gather*}
$$

The eigenfunctions can be defined

$$
\begin{equation*}
\psi_{B, n}(r)=J_{0}\left(\frac{\beta_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{n}\right)}{Y_{1}\left(\beta_{n}\right)} Y_{0}\left(\frac{\beta_{n}}{r_{4}} r\right), \quad \mathrm{r}_{1} \leq r \leq r_{4} \tag{9}
\end{equation*}
$$

where $\frac{\beta_{n}}{r_{4}}$ is the wavenumber on the radial direction which satisfies the rigid wall boundary condition on the inner face of the cylinder,

$$
\begin{equation*}
\left.\frac{\partial \psi_{B, n}(r)}{\partial r}\right|_{r=r_{1}}=0 \tag{10}
\end{equation*}
$$

On the annular rigid duct, the acoustic pressure and axial velocity are similar to those in the inlet pipe, and it can be expressed

$$
\begin{gather*}
P_{D}(r, x)=\sum_{n=0}^{\infty}\left(D_{n}^{+} e^{-j k_{x, D, n}\left(x-l_{C}\right)}+D_{n}^{-} e^{j k_{x, D, n}\left(x-l_{C}\right)}\right) \psi_{D, n}(r), \\
\mathrm{u}_{\mathrm{x}, \mathrm{D}}(\mathrm{r}, \mathrm{x})=\frac{1}{\rho_{0} \omega} \sum_{n=0}^{\infty} k_{x, D, n}\left(D_{n}^{+} e^{-j k_{x, D, n}\left(x-l_{C}\right)}-D_{n}^{-} e^{j k_{x, D, n}\left(x-l_{C}\right)}\right) \psi_{D, n}(r), \\
\psi_{D, n}(r)=J_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{n}\right)}{Y_{1}\left(\lambda_{n}\right)} Y_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right), \quad \mathrm{r}_{2} \leq r \leq r_{4}, \tag{11}
\end{gather*}
$$

where $\frac{\lambda_{n}}{r_{4}}$ is the wavenumber on the radial direction which satisfies the rigid wall boundary condition on the inner face of the cylinder,

$$
\begin{equation*}
\left.\frac{\partial \psi_{D, n}(r)}{\partial r}\right|_{r=r_{2}}=0 \tag{12}
\end{equation*}
$$

The sound propagation in the dissipative expansion chamber is also governed by

$$
\frac{\partial^{2} P}{\partial r^{2}}+\frac{1}{r} \frac{\partial P}{\partial r}+\frac{\partial^{2} P}{\partial x^{2}}+\kappa^{2} P=0, \quad \kappa=\left\{\begin{array}{ll}
k_{0} & 0 \leq \mathrm{r} \leq \mathrm{r}_{3}  \tag{13}\\
\widetilde{k} & \mathrm{r}_{3} \leq \mathrm{r} \leq \mathrm{r}_{4}
\end{array},\right.
$$

with $\tilde{k}$ denoting the wavenumber of the fibrous material. The solutions for the acoustic pressure and the particle velocity in x direction may be expressed

$$
\begin{gather*}
P_{C}(r, x)=\sum_{n=0}^{\infty}\left(C_{n}^{+} e^{-j k_{x, C, n} x}+C_{n}^{-} e^{j k_{x, C, n} x}\right) \begin{cases}\psi_{C 1, n}(r) & 0 \leq \mathrm{r} \leq \mathrm{r}_{3} \\
\psi_{C 2, n}(r) & \mathrm{r}_{3} \leq \mathrm{r} \leq \mathrm{r}_{4}\end{cases} \\
u_{C, n}=\frac{1}{\rho_{0} \omega} \sum_{n=0}^{\infty} k_{x, C, n}\left(C_{n}^{+} e^{-j k_{x, C, n} x}-C_{n}^{-} e^{j k_{x, C, n} x}\right) \begin{cases}\psi_{C 1, n}(r) & 0 \leq \mathrm{r} \leq \mathrm{r}_{3} \\
\frac{\rho_{0}}{\tilde{\rho}} \psi_{C 2, n}(r) & \mathrm{r}_{3} \leq \mathrm{r} \leq \mathrm{r}_{4}\end{cases} \tag{14}
\end{gather*}
$$

$\psi_{C 1, n}(r)$ and $\psi_{c 2, n}(r)$ are the eigenfunctions for air and fiber, respectively. $k_{x, B, n}$ is being the common wavenumber in the axial direction for both the air and fiber. The eigenfunctions are

$$
\begin{gather*}
\psi_{C 1, n}(r)=J_{0}\left(\frac{\chi_{n}}{r_{3}} r\right), \quad 0 \leq \mathrm{r} \leq \mathrm{r}_{3}, \\
\psi_{C 2, n}(r)=C_{1} J_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right)+C_{2} Y_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right), \quad \mathrm{r}_{3} \leq \mathrm{r} \leq \mathrm{r}_{4} . \tag{15}
\end{gather*}
$$

The coefficients of $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are obtained by using the boundary conditions on the radial direction expressed as

$$
\begin{equation*}
\left.\frac{\partial \psi_{C 2, n}(r)}{\partial r}\right|_{r=r_{4}}=0,\left.\quad \psi_{C 1, n}(r)\right|_{r=r_{3}}=\left.\psi_{C 2, n}(r)\right|_{r=r_{3}},\left.\quad \frac{1}{\rho_{0}} \frac{\partial \psi_{C 1, n}(r)}{\partial r}\right|_{r=r_{3}}=\left.\frac{1}{\widetilde{\rho}} \frac{\partial \psi_{C 2, n}(r)}{\partial r}\right|_{r=r_{3}} . \tag{16}
\end{equation*}
$$

After some manipulations and rearrangements, the characteristic equation for the expansion chamber can be obtained as

$$
\begin{equation*}
\frac{\rho_{0} \tilde{\chi}_{n} \cdot r_{2} \cdot J_{0}\left(\chi_{n}\right)}{\tilde{\rho} \cdot \chi_{n} \cdot r_{3} \cdot J_{1}\left(\chi_{n}\right)}=\frac{J_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r_{3}\right) Y_{1}\left(\tilde{\chi}_{n}\right)-Y_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r_{3}\right) J_{1}\left(\tilde{\chi}_{n}\right)}{J_{1}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r_{3}\right) Y_{1}\left(\tilde{\chi}_{n}\right)-Y_{1}\left(\frac{\widetilde{\chi}_{n}}{r_{4}} r_{3}\right) J_{1}\left(\tilde{\chi}_{n}\right)} \tag{17}
\end{equation*}
$$

with $Y_{0}, Y_{1}$ and $J_{1}$ being the zeroth order Neumann, first order Neumann, and first order Bessel functions, respectively. The radial wavenumbers in air and fiber are given as

$$
\begin{equation*}
\frac{\chi_{n}}{r_{3}}=\sqrt{k_{0}^{2}-k_{x, B, n}^{2}} \text { (air), } \quad \frac{\widetilde{\chi}_{n}}{r_{4}}=\sqrt{\widetilde{k}^{2}-k_{x, B, n}^{2}} \text { (fiber), } \tag{18}
\end{equation*}
$$

where $k_{x, B, n}$ is common wavenumber in the axial direction for both the air and fiber. Solving equations (17) and (18) together, the radial wavenumbers can be calculated for the expansion chamber. The boundary condition at the left wall of the chamber is

$$
\begin{equation*}
\left.\mathrm{u}_{\mathrm{x}, \mathrm{~B}}(\mathrm{r}, \mathrm{x})\right|_{\mathrm{x}=-l_{1}}=0 . \tag{19}
\end{equation*}
$$

Similarly, the boundary condition at the right wall of the chamber is

$$
\begin{equation*}
\mathrm{u}_{\mathrm{x}, \mathrm{D}}(\mathrm{r}, \mathrm{x}){\underset{\mathrm{x}=l_{\mathrm{C}}+l_{2}}{ }=0 . . . . ~} \tag{20}
\end{equation*}
$$

With the expressions of pressure and velocity of inlet, outlet, extended chambers and expansion chamber, the unknown coefficients $A_{n}, B_{n}, C_{n}, D_{n}$ and $E_{n}$ are then determined by using the boundary conditions at the expansion $(x=0)$ and contraction $\left(x=1_{c}\right)$. After some more manipulations and arrangements,

$$
\begin{align*}
& \left(A_{s}^{+}+A_{s}^{-}\right) \int_{0}^{r_{1}}\left(J_{0}\left(\frac{\alpha_{s}}{r_{1}} r\right)\right)^{2} r \partial r=\sum_{n=0}^{\infty}\left(C_{n}^{+}+C_{n}^{-}\right) \int_{0}^{r_{1}} J_{0}\left(\frac{\chi_{n}}{r_{3}} r\right) J_{0}\left(\frac{\alpha_{s}}{r_{1}} r\right) r \partial r,  \tag{21}\\
& B_{s}^{-}\left(e^{-2 j k_{x, B, s} l_{1}}+1\right) \int_{r_{1}}^{r_{4}}\left(J_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{s}\right)}{Y_{1}\left(\beta_{s}\right)} Y_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)\right)^{2} r \partial r=\sum_{n=0}^{\infty}\left(C_{n}^{+}+C_{n}^{-}\right)\left(\int_{r_{1}}^{r_{3}} J_{0}\left(\frac{\chi_{n}}{r_{3}} r\right)\left(J_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{s}\right)}{Y_{1}\left(\beta_{s}\right)} Y_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)\right) r \partial r\right. \\
& \left.+\int_{r_{3}}^{r_{4}} D\left[J_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\tilde{\chi}_{n}\right)}{Y_{1}\left(\widetilde{\chi}_{n}\right)} Y_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right)\right]\left(J_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{s}\right)}{Y_{1}\left(\beta_{s}\right)} Y_{0}\left(\frac{\beta_{s}}{r_{4}} r\right)\right) r \partial r\right),  \tag{22}\\
& \sum_{n=0}^{\infty} k_{x, A, n}\left(A_{n}^{+}-A_{n}^{-}\right) \int_{0}^{r_{1}} J_{0}\left(\frac{\alpha_{n}}{r_{1}} r\right) J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right) r \partial r+\sum_{n=0}^{\infty} k_{x, B, n} B_{n}^{-}\left(e^{-2 j k_{x, B, n} l_{1}}-1\right)\left(\int_{r_{1}}^{r_{3}}\left(J_{0}\left(\frac{\beta_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{n}\right)}{Y_{1}\left(\beta_{n}\right)} Y_{0}\left(\frac{\beta_{n}}{r_{4}} r\right)\right) J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right) r \partial r \\
& \left.+\int_{r_{3}}^{r_{4}}\left(J_{0}\left(\frac{\beta_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\beta_{n}\right)}{Y_{1}\left(\beta_{n}\right)} Y_{0}\left(\frac{\beta_{n}}{r_{4}} r\right)\right) D\left[J_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\tilde{\chi}_{s}\right)}{Y_{1}\left(\widetilde{\chi}_{s}\right)} Y_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)\right] r \partial r\right) \\
& =k_{x, C, s}\left(C_{s}^{+}-C_{s}^{-}\right)\left(\int_{0}^{r_{1}}\left(J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right)^{2} r \partial r+\int_{r_{1}}^{r_{3}}\left(J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right)^{2} r \partial r+\frac{\rho_{0}}{\tilde{\rho}} \int_{r_{3}}^{r_{4}}\left(D\left[J_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\widetilde{\chi}_{s}\right)}{Y_{1}\left(\widetilde{\chi}_{s}\right)} Y_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)\right]\right)^{2} r \partial r\right) \text {, }  \tag{23}\\
& \left(E_{s}^{+}+E_{s}^{-}\right) \int_{0}^{r_{2}}\left(J_{0}\left(\frac{\alpha_{s}}{r_{2}} r\right)\right)^{2} r \partial r=\sum_{n=0}^{\infty}\left(C_{n}^{+} e^{-j k_{x, C, n} l_{C}}+C_{n}^{-} e^{j k_{x, C, n}}\right) \int_{0}^{r_{2}} J_{0}\left(\frac{\chi_{n}}{r_{3}} r\right) J_{0}\left(\frac{\alpha_{s}}{r_{2}} r\right) r \partial r,  \tag{24}\\
& D_{s}^{+}\left(1+e^{-2 j k_{x, D, s} l_{2}}\right) \int_{r_{2}}^{r_{4}}\left(J_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{s}\right)}{Y_{1}\left(\lambda_{s}\right)} Y_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right)\right)^{2} r \partial r \\
& =\sum_{n=0}^{\infty}\left(C_{n}^{+} e^{-j k_{x, C, n} l_{C}}+C_{n}^{-} e^{j k_{x, C, n} l_{C}}\right)\left(\int_{r_{2}}^{r_{3}} J_{0}\left(\frac{\chi_{n}}{r_{3}} r\right)\left(J_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{s}\right)}{Y_{1}\left(\lambda_{s}\right)} Y_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right)\right) \gamma \partial r\right. \\
& +\int_{r_{3}}^{r_{4}} D\left[J_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\tilde{\chi}_{n}\right)}{Y_{1}\left(\tilde{\chi}_{n}\right)} Y_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r\right)\right]\left(J_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{s}\right)}{Y_{1}\left(\lambda_{s}\right)} Y_{0}\left(\frac{\lambda_{s}}{r_{4}} r\right) r \partial r\right), \tag{25}
\end{align*}
$$

$$
\begin{align*}
& \sum_{n=0}^{\infty} k_{x, E, n}\left(E_{n}^{+}-E_{n}^{-}\right) \int_{0}^{r_{2}} J_{0}\left(\frac{\alpha_{n}}{r_{2}} r\right) J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right) r \partial r+\sum_{n=0}^{\infty} k_{x, D, n} D_{n}^{+}\left(1-e^{-2 j k_{x, D, n} l_{2}}\right)\left(\int_{r_{2}}^{r_{3}}\left(J_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{n}\right)}{Y_{1}\left(\lambda_{n}\right)} Y_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right)\right) J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right) r \partial r \\
& \left.+\int_{r_{3}}^{r_{4}}\left(J_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right)-\frac{J_{1}\left(\lambda_{n}\right)}{Y_{1}\left(\lambda_{n}\right)} Y_{0}\left(\frac{\lambda_{n}}{r_{4}} r\right)\right) D\left[J_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\tilde{\chi}_{s}\right)}{Y_{1}\left(\tilde{\chi}_{s}\right)} Y_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)\right] r \partial r\right) \\
& =k_{x, C, s}\left(C_{n}^{+} e^{-j k_{x, C, n} l_{C}}-C_{n}^{-} e^{i k_{x, C, n} l_{C}}\right)\left(\int_{0}^{r_{2}}\left(J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right)^{2} r \partial r+\int_{r_{2}}^{r_{3}}\left(J_{0}\left(\frac{\chi_{s}}{r_{3}} r\right)\right)^{2} r \partial r+\frac{\rho_{0}}{\tilde{\rho}} \int_{r_{3}}^{r_{4}}\left(D\left[J_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)-\frac{J_{1}\left(\tilde{\chi}_{s}\right)}{Y_{1}\left(\tilde{\chi}_{s}\right)} Y_{0}\left(\frac{\tilde{\chi}_{s}}{r_{4}} r\right)\right]\right)^{2} r \partial r\right) \text {, } \\
& D=\frac{J_{0}\left(\chi_{n}\right) \cdot Y_{1}\left(\tilde{\chi}_{n}\right)}{J_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r_{3}\right) \cdot Y_{1}\left(\widetilde{\chi}_{n}\right)-J_{1}\left(\widetilde{\chi}_{n}\right) \cdot Y_{0}\left(\frac{\tilde{\chi}_{n}}{r_{4}} r_{3}\right)}, \tag{26}
\end{align*}
$$

In order to determine the transmission loss of the circular expansion chamber including fiber material and extended inlet/outlet tubes, it is assumed that: (1) the incoming wave is planar and its magnitude is set to be unity, (2) an anechoic termination is imposed at the exit of the chamber by setting the reflected wave to zero. Thus, equations (21) - (26) can give a large (theoretically infinite) number of relations, 6(s+1) for a large number of unknowns, 6(n+1). The unknowns are the pressure magnitudes for incident and reflected waves in the regions A, $\mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . Because higher modes have a diminishing effect on the solution, s and n can be truncated to $\eta$ resulting in $6(\eta+1)$ equations with $6(\eta+1)$ unknowns. For the geometries and frequencies investigated here, $\eta=5$ were found to be sufficient. So that, after some more manipulations and arrangements, the transmission loss is determined in the center of the outlet duct by

$$
\begin{equation*}
T L=-20 \log _{10}\left|\left(a_{2} / a_{1}\right) \sum_{n=0}^{\eta} E_{n}^{+} e^{-j k_{x, E, n} l_{2}}\right| \tag{27}
\end{equation*}
$$

## NUMERICAL EVALUATIONS

This section is devoted to the numerical evaluation of the expressions obtained above. In order to investigate the transmission loss, the characteristic impedance $\tilde{Z}$ and the wavenumber $\widetilde{k}$ of the absorbing material is given, as in reference [1]

$$
\begin{equation*}
\frac{\widetilde{Z}}{Z_{0}}=\left(1+0.0855 .\left(\frac{f}{R}\right)^{-0.754}\right)+j .\left(-0.0765 .\left(\frac{f}{R}\right)^{-0.732}\right), \quad \frac{\widetilde{k}}{k_{0}}=\left(1+0.1472 .\left(\frac{f}{R}\right)^{-0.577}\right)+j .\left(-0.1734 .\left(\frac{f}{R}\right)^{-0.595}\right) \tag{28}
\end{equation*}
$$

where $Z_{0}=\rho_{0} c_{0}$ being the characteristic impedance of the air and $f(\mathrm{~Hz})$ denotes frequency and R (mks Rayls $/ \mathrm{m}$ ) the resistivity of the absorbing material. In figure 2, the transmission loss from the analytical approach is compared with FEM prediction calculated in Msc.Actran. The two curves are in good agreement. Figure 3 expresses the analytical transmission loss results for the different fibers. Higher fiber resistivities lead to higher transmission loss at medium to high frequencies, while the lower resistivities improve transmission loss at low frequencies. Figure 4 examines the effect of variation of fiber thickness, on the transmission loss. The results for the empty silencer are also included. The overall performance of the silencer is generally improved by increasing fiber thickness. Thus, for maximum attenuation, the diameters for the airway and the inlet/outlet pipes would be the same. Figure 5 shows the transmission loss results for different extended tube lengths. The results for non-extended tubes are also included. Especially, at low and middle frequencies the overall performance of the silencer is generally improved by increasing lengths of extended tube.


Figure 2-Transmission loss of circular expansion chamber including fiber material and extended inlet/outlet tubes. $\left(r_{1}=0.0245 \mathrm{~m}, r_{2}=0.0245 \mathrm{~m}, r_{3}=0.075 \mathrm{~m}, r_{4}=0.0822 \mathrm{~m}, L=0.2572 \mathrm{~m}, l_{1}=0.06 \mathrm{~m}, l_{2}=\right.$ $0.03 \mathrm{~m}, \mathrm{R}=4896$ Rayls $/ \mathrm{m}$ )


Figure 3-Transmission loss for different fiber resistivities. $\left(r_{1}=0.0245 \mathrm{~m}, r_{2}=0.0245 \mathrm{~m}, r_{3}=0.035 \mathrm{~m}\right.$, $\left.r_{4}=0.0822 \mathrm{~m}, L=0.2572 \mathrm{~m}, l_{l}=0.06 \mathrm{~m}, l_{2}=0.03 \mathrm{~m}\right)$


Figure 4-Transmission loss for different fiber thicknesses. $\left(r_{1}=0.0245 \mathrm{~m}, r_{2}=0.0245 \mathrm{~m}, r_{4}=0.0822 \mathrm{~m}\right.$, $L=0.2572 \mathrm{~m}, l_{1}=0.06 \mathrm{~m}, l_{2}=0.03 \mathrm{~m}, R=4896$ Rayls $/ \mathrm{m}$ )


Figure 5-Transmission loss for different extended tube lengths. $\left(r_{1}=0.0245 \mathrm{~m}, r_{2}=0.0245 \mathrm{~m}, r_{3}=\right.$ $0.035 m, r_{4}=0.0822 m, L=0.2572 m, R=4896$ Rayls $/ m$ )

## CONCLUSIONS

This study addresses essentially to the investigation of the acoustic attenuation performance of a circular expansion chamber including fiber material and extended inlet/outlet tubes, in which a theoretical analysis has been made in order to determine the transmission loss of the system as the acoustic performance parameter. First, a two dimensional analytical solution for the transmission loss is established via boundary value formulation for the concentric configuration of the circular expansion chamber having fiber material and extended tubes. It is important to note that the solutions obtained are expressed in terms of parameters characterizing the physical properties of the system under consideration. In order to compare the solutions, transmission loss of the system is also computed by using the finite element method model. All equations are then numerically solved for various combinations of physical parameters and the results are represented in figures. The comparison of the numerical results obtained via analytical formulations justifies the finite element method approach.

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