DYNAMIC MODELING AND MEASUREMENTS ON A RECIPROCATING HERMETIC COMPRESSOR

H. EROL[1], T. DURAKBAŞA[2] and H.T. BELEK[3]

[1]Research Assistant, Istanbul Technical University, Faculty of Mechanical Engineering, [2]Research Specialist, Arçelik A.Ş., R&D Department, Çayırova 41460 İstanbul,Turkey, [3]Professor, Istanbul Technical University, Faculty of Mechanical Engineering, Gümüşsuyu 80191 İstanbul Turkey

ABSTRACT

In the present study a non-linear dynamic model of the compressor is developed to predict the entire motion of the compressor. The motion is described in terms of three linear positional coordinates (X, Y, Z) , four Euler Parameters (θ_0, θ_1, θ_2, θ_3) and the position of the crankshaft β. The model has seven degrees of freedom since the four Euler Parameters are related by a single equation. A computer program is prepared for simulation of the motion. The crankshaft speed of rotation and the time response of the compressor unit during transient and steady state motion are predicted. The model is initially tested in a limited experiment to predict the peak displacements of the compressor body in three directions and good agreement was observed.

1. INTRODUCTION

The growing demand in the market for highly efficient, more reliable and less expensive compressors have activated the manufacturers to develop state-of-the-art analytical tools to predict, evaluate and optimize the existing as well as creating the new designs.

Any improvement in the compressor design requires an in-depth knowledge of the compressor unit. Dynamic behavior of the unit during the transient start-up and shut-down periods needs to be studied carefully at the design stage. Development of a mathematical tool for predicting the dynamic behavior saves considerable time and reduces the unnecessary number of prototypes as well as the development costs.

The main components of a hermetic compressor are the hermetic casing on the rubber isolators, the stator-rotor assembly suspended by springs and a flexible discharge pipe, and the slider-crank mechanism. Studies have been conducted to understand the dynamic characteristics of these components, [1-7]. In a recent study, Dufour et. al. [8] developed a linear model to predict the dynamic behavior of a single cylinder refrigerant compressor during the start-up, steady-state and shut-down motions. Instantaneous speed of rotation of the crankshaft, and displacement and acceleration of the compressor unit can also be predicted.

In all the studies conducted so far, the coefficient matrices in the classical differential system of equations have been considered constant by assuming a small displacement of the center of inertia. Although this assumption may be correct during the steady-state motion, from experimental observations it is a well known fact that the compressor body within the hermetic casing may execute substantial amount of motion during the start-up and shut-down motions.

In the present study, a seven degree of freedom non-linear theoretical model is developed where the mass, stiffness and damping matrices are function of time and the start-up, steady-state, shut-down motions as well as the instantaneous speed of rotation of the crankshaft and the vibration amplitudes of the compressor unit can also be predicted. The model is initially tested in a limited experiment to predict the peak displacements of the compressor body in three directions and a good agreement is observed.

2. MATHEMATICAL MODELLING

The main assumptions made in the present study to develop the equations of motion can be summarizes as follows: (i) The motion of the hermetic housing is neglected, (ii) stator body, piston, connecting rod and crankshaft are assumed to be rigid, (iii) the discharge line and suspension springs are massless, deformable bodies having only elastic and damping effects, (iv) the effect of the discharge pressure within the discharge line is omitted, (v) all body forces due to the gravity field are omitted, (vi) the compression and expansion processes are expressed as polytropic processes and discharge process is expressed as constant pressure process, (vii) the compressor motor drive characteristics is approximated using the curve fitted to the measured motor torque-speed curve, (viii) at the start-up instant the equivalent resistive torque acting on the crank shaft is calculated from the steady state P-V diagram, (ix) at the shut-down instant the driving moment is ideally equated to zero.

The cross-sectional view and the reference frames of a typical single cylinder compressor are illustrated Ir Figure 1. The compressor system analyzed consists of a stator frame, rotor, crankshaft, connecting rod, piston and mounting springs. The fixed inertial coordinate system (OXYZ) is placed at the equilibrium position of the center of the frame of the slider-crank mechanism. The motion of the moving frame, ($CX_CY_CZ_C$), which is connected to the center of rotation of the crank C, is calculated with respect to the inertial coordinate system (OXYZ). It is assumed that both frames of reference coincide initially.

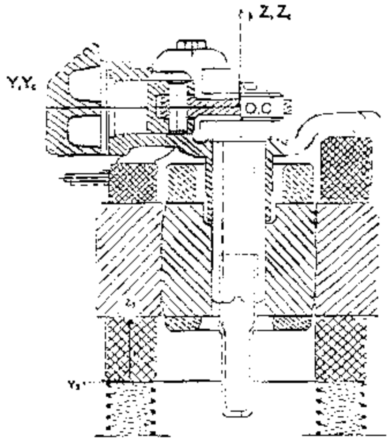

Fig.1. The reference frames of a typical single cylinder compressor.

The mathematical model developed predicts the transient and steady-state motion of a point on the compressor. The motion of such a point is described in terms of three linear positional coordinates (X,Y,Z), four Euler parameters (θ_0, θ_1, θ_2, θ_3) as the angular position coordinates and the angular position of the crankshaft, β. Since the four Euler parameters are related to each by a single equation, the compressor system has seven degrees of freedom.

The schematic view of the slider-crank mechanism is given in Figure 2.

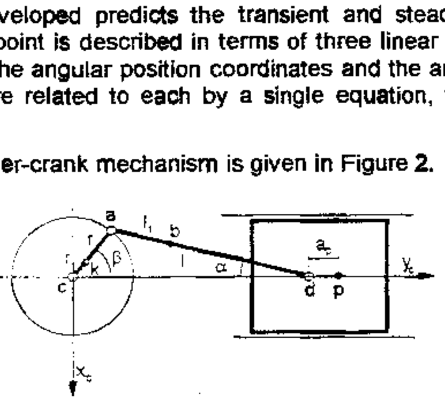

Fig.2. The slider-crank mechanism.

For a n-degree of freedom system, if ψ number of constraint equations exist between the generalized coordinates, the equation of motion can be developed by using the "Method of Lagrange Multipliers". In this formulation Lagrange Equations are given by the following formula:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}+\frac{\partial D}{\partial \dot{q}_i}=Q_i+\sum_{k=1}^{\psi}\sigma_k\frac{\partial f_k}{\partial q_i},\quad i=1,..\ n+\psi \tag{1}$$

Where
T = kinetic energy of the system,
V = potential energy of the system,
D = dissipated energy of the system,
q_i = generalized coordinates,
Q_i = generalized forces with respect to q_i generalized coordinates,
σ_k = Lagrange multipliers,
f_k = Constrained equations.

In equation (1) the total kinetic energy, potential energy and the dissipated energy expressions of the system components should be written with respect to the inertial frame of reference. Those expressions are given as follows: Total kinetic energy of the system can be expressed as:

$$
\begin{aligned}
T = \frac{1}{2}\Big[& \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_k^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_k^c\right)^T \mathbf{m}_k \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_k^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_k^c\right) + \left(\omega_c^o + \mathbf{A}\mathbf{D}\omega_k^c\right)^T \mathbf{A}\mathbf{D}\mathbf{I}_k\mathbf{D}^T\mathbf{A}^T\left(\omega_c^o + \mathbf{A}\mathbf{D}\omega_k^c\right) \\
& + \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_b^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_b^c\right)^T \mathbf{m}_b \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_b^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_b^c\right) + \left(\omega_c^o + \mathbf{A}\mathbf{B}\omega_b^c\right)^T \mathbf{A}\mathbf{B}\mathbf{I}_b\mathbf{B}^T\mathbf{A}^T\left(\omega_c^o + \mathbf{A}\mathbf{B}\omega_b^c\right) \\
& + \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_p^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_p^c\right)^T \mathbf{m}_p \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_p^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_p^c\right) + \omega_c^{o\,T} \mathbf{A}\mathbf{I}_p\mathbf{A}^T\omega_c^o \\
& + \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_r^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_r^c\right)^T \mathbf{m}_r \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_r^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_r^c\right) + \left(\omega_c^o + \mathbf{A}\mathbf{D}\omega_r^c\right)^T \mathbf{A}\mathbf{D}\mathbf{I}_r\mathbf{D}^T\mathbf{A}^T\left(\omega_c^o + \mathbf{A}\mathbf{D}\omega_r^c\right) \\
& + \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_{cw}^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_{cw}^c\right)^T \mathbf{m}_{cw} \left(\dot{\mathbf{r}}_c^o + \mathbf{A}\dot{\mathbf{r}}_{cw}^c + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_{cw}^c\right) + \left(\dot{\mathbf{r}}_c^o + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_g^c\right)^T \mathbf{m}_k \left(\dot{\mathbf{r}}_c^o + \tilde{\omega}_c^o \mathbf{A}\mathbf{r}_g^c\right) + \omega_c^{o\,T} \mathbf{A}\mathbf{I}_g\mathbf{A}^T\omega_c^o\Big]
\end{aligned}
\tag{2}
$$

The total potential energy of the system can be expressed as:

$$V = \frac{1}{2}\sum_{i=1}^{5} \mathbf{r}_{y_i}^{o\,T}\mathbf{k}_{y_i}^o\mathbf{r}_{y_i}^o \tag{3}$$

The total dissipated energy of the system,

$$D = \frac{1}{2}\sum_{i=1}^{5}\left(\dot{\mathbf{r}}_{y_i}^{o\,T}\mathbf{C}_{y_i}^o\dot{\mathbf{r}}_{y_i}^o\right) + \mathbf{C}_d^o \tag{4}$$

In equation (2-4) **A** denotes the transformation matrix in terms of the Euler parameters. The subscripts k, b, p, cw, r, g, y and d denotes crank, connecting rod, piston, counterweight, rotor, stator, springs and damping due to oil bath respectively. The super-scripts "o" and "c" represent the fixed inertial coordinate system and the moving coordinate system respectively.

As for the damping, main contribution is due to the oil bath within the hermetic housing, it is also assumed that the damping existed in the suspension system of the compressor. Experiments were performed to determine the frequency response function of the compressor in the oil bath at stand still. From the natural frequencies of the system modal viscous damping coefficients were extracted. These values were used in the second term of equation (4) with appropriate transformations.

Inserting the expressions (2), (3) and (4) into equation (1) yields a set of differential equations (5) describing the motion of the compressor

$$
\begin{aligned}
& a_{i,1}\ddot{x} + a_{i,2}\ddot{y} + a_{i,3}\ddot{z} + a_{i,4}\ddot{\theta}_0 + a_{i,5}\ddot{\theta}_1 + a_{i,6}\ddot{\theta}_2 + a_{i,7}\ddot{\theta}_3 + a_{i,8}\ddot{\beta} + a_{i,9}\dot{\beta}^2 + a_{i,10}\dot{\theta}_0\dot{\beta} \\
& + a_{i,11}\dot{\theta}_1\dot{\beta} + a_{i,12}\dot{\theta}_2\dot{\beta} + a_{i,13}\dot{\theta}_3\dot{\beta} + a_{i,14}\dot{x}\dot{\theta}_0 + a_{i,15}\dot{x}\dot{\theta}_1 + a_{i,16}\dot{x}\dot{\theta}_2 + a_{i,17}\dot{x}\dot{\theta}_3 + a_{i,18}\dot{y}\dot{\theta}_0 \\
& + a_{i,19}\dot{y}\dot{\theta}_1 + a_{i,20}\dot{y}\dot{\theta}_2 + a_{i,21}\dot{y}\dot{\theta}_3 + a_{i,22}\dot{z}\dot{\theta}_0 + a_{i,23}\dot{z}\dot{\theta}_1 + a_{i,24}\dot{z}\dot{\theta}_2 + a_{i,25}\dot{z}\dot{\theta}_3 + a_{i,26}\dot{\theta}_0^2 \\
& + a_{i,27}\dot{\theta}_0\dot{\theta}_1 + a_{i,28}\dot{\theta}_0\dot{\theta}_2 + a_{i,29}\dot{\theta}_0\dot{\theta}_3 + a_{i,30}\dot{\theta}_1^2 + a_{i,31}\dot{\theta}_1\dot{\theta}_2 + a_{i,32}\dot{\theta}_1\dot{\theta}_3 + a_{i,33}\dot{\theta}_2^2 + a_{i,34}\dot{\theta}_2\dot{\theta}_3 \\
& + a_{i,35}\dot{\theta}_3^2 + a_{i,36}\dot{x} + a_{i,37}\dot{y} + a_{i,38}\dot{z} + a_{i,39}\dot{\theta}_0 + a_{i,40}\dot{\theta}_1 + a_{i,41}\dot{\theta}_2 + a_{i,42}\dot{\theta}_3 + a_{i,43}\dot{\beta} + a_{i,44}x \\
& + a_{i,45}y + a_{i,46}z + a_{i,47}\theta_0 + a_{i,48}\theta_1 + a_{i,49}\theta_2 + a_{i,50}\theta_3 = f_i(t) + 2\sigma\theta_{i-4}
\end{aligned}
\tag{5}
$$

$$i = 1, \ldots, 8$$

These eight nonlinear differential equations of the compressor system can be expressed in matrix form as follows:

$$\mathbf{m}\ddot{\mathbf{q}} + \mathbf{c}\dot{\mathbf{q}} + \mathbf{k}\mathbf{q} = \mathbf{f}(t) + 2\sigma\mathbf{p} \tag{6}$$

The inertial terms of the moving parts expressed by the elements of the mass matrix of the slider-crank mechanism depend on the crankshaft position, β, are hence functions of time.

The relationship between the Euler Parameters is given by

$$\theta_0^2+\theta_1^2+\theta_2^2+\theta_3^2=1 \tag{7}$$

The resulting system comprises eight differential equations plus an algebraic equation which should be solved simultaneously. The system of equations can be put into the following matrix form suitable for a step-by-step numerical integration procedure:

$$\ddot{\mathbf{q}}=-\mathbf{m}^{-1}\mathbf{c}\dot{\mathbf{q}}-\mathbf{m}^{-1}\mathbf{k}\mathbf{q}+\mathbf{m}^{-1}\mathbf{f}(t)-2\mathbf{T}\dot{\mathbf{p}}^{T}\dot{\mathbf{q}} \tag{8}$$

Equation (8) further can be transformed into the following form:

$$\dot{\mathbf{x}}=\mathbf{E}\mathbf{x}+\mathbf{Q} \tag{9}$$

where

$$\mathbf{E}=\begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{m}^{-1}\mathbf{k} & -\mathbf{m}^{-1}\mathbf{c}-2\mathbf{T}\dot{\mathbf{p}}^{T} \end{bmatrix} \quad \mathbf{Q}=\begin{Bmatrix} 0 \\ \mathbf{m}^{-1}\mathbf{f}(t) \end{Bmatrix}, \quad \mathbf{x}=\begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix}$$

Slider-crank mechanism is considered as a system upon which the external forces act. These forces included in the force vector $\mathbf{f}(t)$are: (i) The electromagnetic moment generated between the stator and the rotor which is the function of speed. This moment is expressed analytically by an empirical formula generated from the experimentally determined moment characteristics of the motor. (ii) Gas forces on the piston during the suction and compression are expressed as function of the crank angle by assuming polytropic compression and suction in a similar manner as suggested in reference [3]. (iii) Frictional forces are modelled by considering the Newton's formula which relates dynamic viscosity, area of the sliding surfaces, relative velocity and clearance between the sliding surfaces to the frictional force.

3. THEORETICAL and EXPERIMENTAL RESULTS

The theoretical model is applied on a real life single cylinder compressor commonly used in the refrigerators. All the physical model parameters that are used as input in the computer code are determined for the relevant components of the compressor. Position of the crankshaft at t=0 is also given as an initial condition. Runge-Kutta-Merson method is then utilized in the step by step numerical solution of equation (9). Vibration amplitudes in the direction along with the piston movement (Y), perpendicular to the piston movement (X) and in the vertical direction to XY plane (Z) are calculated at the transducer location. Some of the theoretical results in the form of rotational speed and vibration amplitudes versus time are given in Figures 3 - 6. Peak displacement values in these plots will be compared to the experimentally obtained peak vibrational displacements.

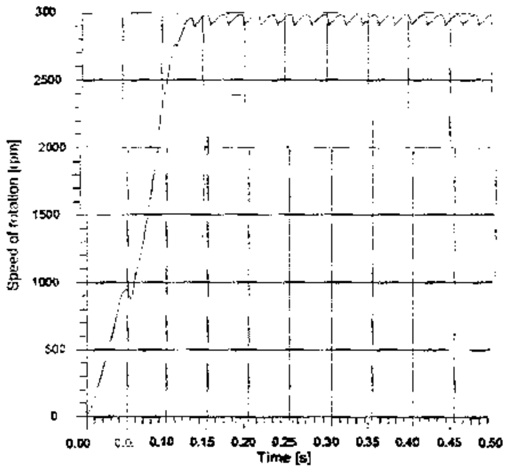

Figure 3. Speed of rotation versus time (Start-up).

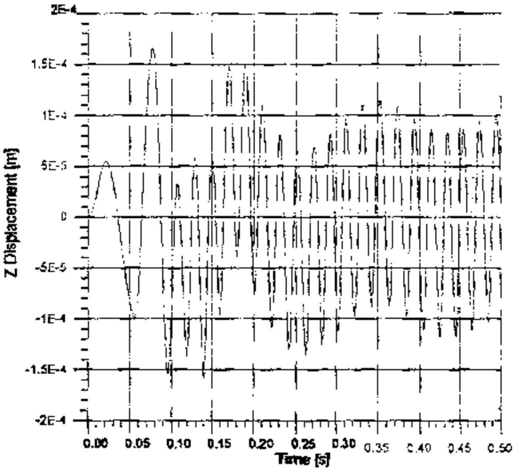

Figure 4. Z displacement at Start-up.

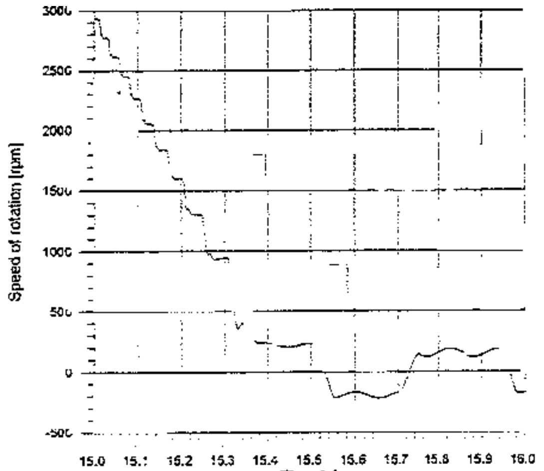

Figure 5. Speed of rotation versus time at shut-down.

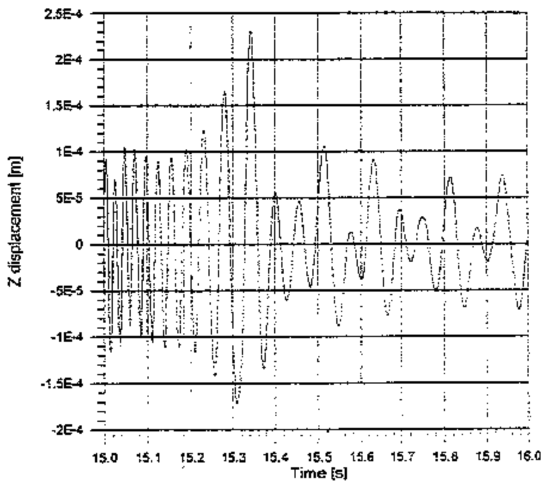

Figure 6. Z displacement at shut-down.

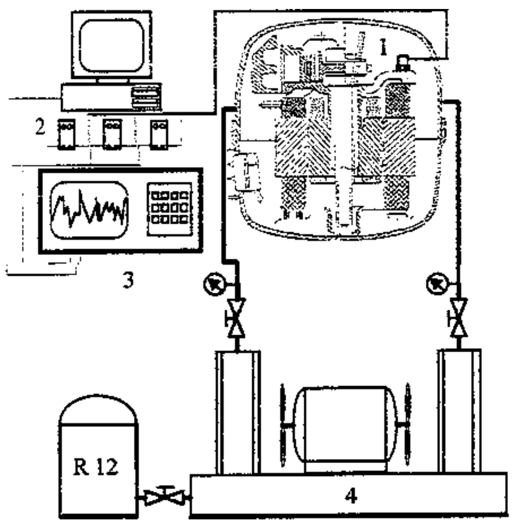

Figure 7. Experimental set-up. (1) Tri-axial accelerometer, (2) Charge amplifiers, (3) Four Channel Oscilloscope with memory, (4) Compressor loading system.

In order to verify the mathematical model an experimental set-up is prepared. The initial set-up is composed of a tri-axial accelerometer located on the stator, charge amplifiers, a four channel oscilloscope with memory and the compressor loading system. A computer is also included for the permanent digital data storage. Pressure fluctuations are recorded at the steady-state operation by a 0.4 gr pressure transducer capable of recording maximum pressure of 250 psia and operating at temperatures between -55 °C to 235 °C. Pressure transducer is carefully mounted to record the pressure within the cylinder head. Using this data the steady state PV-diagram is constructed and the pressure variation within the cylinder head is obtained as function of the crank angle.

Further modifications are being made in the set-up to identify the angular position of the crank shaft and to record the angular speed of the crank shaft as function of time. With these modifications the theoretical model results can be compared with the experimental findings as function of time. At present only the peak experimental amplitudes are compared with model results in the form of order of magnitude as shown in Table 1 to give an idea on the performance of the model.

Table 1. Experimental anc theoretical values for the peak displacements of a point on the compressor. (mm)

Measurement Direction		x	y	z
Start-up	Computed	10.0	7.5	1.5
Start-up	Measured	8	7	2
Steady-state	Computed	0.08	0.45	0.12
Steady-state	Measured	0.10	0.75	0.15
Shut-down	Computed	4.5	3	2
Shut-down	Measured	5.0	5	2.5

4. CONCLUSIONS

A non-linear model with seven degrees of freedom has been developed and applied on a small reciprocating compressor. Under idealised conditions, as stated in the basic assumptions, the numerical response of the model gives reasonable results for the entire motion of the compressor from start-up to shut-down. The speed of the compressor reaches the steady state regime within 0.15 seconds. At steady state, speed fluctuates between 2902-2983 rpm. During the shut-down, speed falls down to zero in approximately 0.5 seconds. During the transient stages computed amplitudes in the horizontal plane lie between 3-10.5 mm. At steady state computed amplitudes are less than 0.5 mm.

An initial experimental set-up is constructed to measure the peak vibration amplitudes during the entire motion of the compressor. Measurements were made at the top of stator body in three directions. Since there exists basic differences between the assumptions and the real system operation, theoretical and experimental results cannot be compared on the same time axis. However, the theoretical model is based on some simplifications. For example, the pressure build-up and decay during start-up and shut-down is neglected, variation of the moment-speed characteristics due to heating in the stator is not taken into account. Only the peak displacements are compared to give an idea if the order of magnitude of the model response is acceptable. It is found that the model response is yielding logical values as for the peak values. Further work is under progress to modify the experimental set-up and to develop the model to compare the results as function of time.

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