A Compressive Sensing Framework for Multirate Signal Estimation

Ender M. Ekşioğlu, A. Korhan Tanc and Ahmet H. Kayran

Istanbul Technical University

Electronics and Communications Engineering Department

Main Headings

Main Headings

- Introduction

Main Headings

- Introduction
- Multirate Signal Estimation Problem

Main Headings

- Introduction
- Multirate Signal Estimation Problem
- Compressive Sensing Prior

Main Headings

- Introduction
- Multirate Signal Estimation Problem
- Compressive Sensing Prior
- Multirate Observations meet Compressive Sensing

Main Headings

- Introduction
- Multirate Signal Estimation Problem
- Compressive Sensing Prior
- Multirate Observations meet Compressive Sensing
- Numerical Results

Main Headings

- Introduction
- Multirate Signal Estimation Problem
- Compressive Sensing Prior
- Multirate Observations meet Compressive Sensing
- Numerical Results
- Conclusions

Introduction

Introduction

- We consider a signal sensing scheme where the underlying signal is observed through a bank of measurement channels working at differing sampling rates.

Introduction

- We consider a signal sensing scheme where the underlying signal is observed through a bank of measurement channels working at differing sampling rates.
- Here, we consider the case where the underlying signal to be observed through this kind of a mechanism is compressible in some transform domain.

Introduction

- We consider a signal sensing scheme where the underlying signal is observed through a bank of measurement channels working at differing sampling rates.
- Here, we consider the case where the underlying signal to be observed through this kind of a mechanism is compressible in some transform domain.
- Compressive sensing is based on the premise that under the compressibility (sparsity) condition it is possible to reconstruct the signal from a number of measurements far fewer than its dimensionality.

Introduction

- We show the that the multichannel multirate signal acquisition mechanism can actually be thought of as a compressive sensing type data sensing method.

Introduction

- We show the that the multichannel multirate signal acquisition mechanism can actually be thought of as a compressive sensing type data sensing method.
- We present numerical results which confirm that when the signal to be observed through the multichannel multirate system is compressible in the DCT domain, compressive sensing based reconstruction from the measurements works effectively.

Multirate Signal Estimation Problem

Multirate Signal Estimation Problem

- We assume a signal acquisition setting where a directly unobservable message signal $x(n)$ is observed through a bank of K sensors working at individual sampling rates.

Multirate Signal Estimation Problem

- We assume a signal acquisition setting where a directly unobservable message signal $x(n)$ is observed through a bank of K sensors working at individual sampling rates.
- Each sensor bank consists of an FIR filter followed by a downsampler with downsampling ratio, N_k.

Multirate Signal Estimation Problem

- We assume a signal acquisition setting where a directly unobservable message signal $x(n)$ is observed through a bank of K sensors working at individual sampling rates.
- Each sensor bank consists of an FIR filter followed by a downsampler with downsampling ratio, N_k.

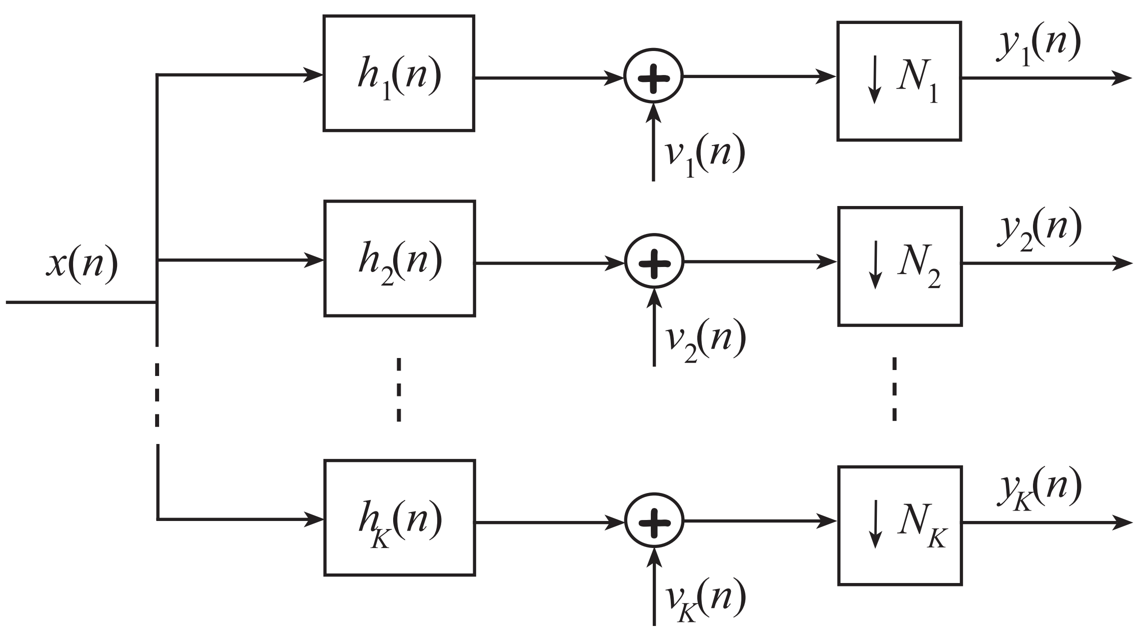

Figure 1: Multirate multichannel signal observation mechanism.

Compressive Sensing Prior Art

Compressive Sensing Prior Art

- Signal processing based on sparse representations has been a subject of active research.

Compressive Sensing Prior Art

- Signal processing based on sparse representations has been a subject of active research.
- A novel signal sensing and reconstruction paradigm based on sparse representation has been developed under the title of "compressive sensing" (or alternately "compressive sampling").

Compressive Sensing Prior Art

- Signal processing based on sparse representations has been a subject of active research.
- A novel signal sensing and reconstruction paradigm based on sparse representation has been developed under the title of "compressive sensing" (or alternately "compressive sampling").
- For a discrete signal $\boldsymbol{x} \in \mathbb{R}^n$, the compressive sensing (CS) data acquisition step is realized by projecting the signal onto a set of sensing vectors $\left\{\boldsymbol{\phi}_j\right\}_{j=1}^{m}$.

Compressive Sensing Prior Art

- The data acquisition step can be summarized in the form of the underdetermined equation

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the observation vector.

Compressive Sensing Prior Art

- The data acquisition step can be summarized in the form of the underdetermined equation

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} \tag{1}$$

  where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the observation vector.
- The reconstruction part of the compressive sensing paradigm handles the ill-posed inverse problem forming an estimate $\widehat{\boldsymbol{x}}$ based on the observation vector $\boldsymbol{y}$.

Compressive Sensing Prior Art

- The data acquisition step can be summarized in the form of the underdetermined equation

$$y = \Phi x \tag{1}$$

  where $y \in \mathbb{R}^m$ denotes the observation vector.
- The reconstruction part of the compressive sensing paradigm handles the ill-posed inverse problem forming an estimate $\widehat{x}$ based on the observation vector y.
- Under the assumption of a sparsity prior for x, the reconstruction step can be reformatted as an optimization problem.

Compressive Sensing Prior Art

- The data acquisition step can be summarized in the form of the underdetermined equation

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} \tag{1}$$

  where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the observation vector.
- The reconstruction part of the compressive sensing paradigm handles the ill-posed inverse problem forming an estimate $\widehat{\boldsymbol{x}}$ based on the observation vector $\boldsymbol{y}$.
- Under the assumption of a sparsity prior for $\boldsymbol{x}$, the reconstruction step can be reformatted as an optimization problem.
- The assumption is that the signal $\boldsymbol{x}$ has a sparse (or more generally compressible) representation in a transform domain expressed by some basis matrix $\boldsymbol{\Psi}$.

Compressive Sensing Prior Art

- The data acquisition step can be summarized in the form of the underdetermined equation

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} \tag{1}$$

  where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the observation vector.
- The reconstruction part of the compressive sensing paradigm handles the ill-posed inverse problem forming an estimate $\widehat{\boldsymbol{x}}$ based on the observation vector $\boldsymbol{y}$.
- Under the assumption of a sparsity prior for $\boldsymbol{x}$, the reconstruction step can be reformatted as an optimization problem.
- The assumption is that the signal $\boldsymbol{x}$ has a sparse (or more generally compressible) representation in a transform domain expressed by some basis matrix $\boldsymbol{\Psi}$.
- $\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}$ is an S-sparse vector.

Compressive Sensing Prior Art

- Compressive sensing reconstruction procedure boils down to finding

$$\widehat{\boldsymbol{\alpha}} = \operatorname{argmin} \|\boldsymbol{\alpha}\|_0 \text{ subject to } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \qquad (2)$$

Compressive Sensing Prior Art

- Compressive sensing reconstruction procedure boils down to finding

$$\widehat{\boldsymbol{\alpha}} = \operatorname{argmin} \|\boldsymbol{\alpha}\|_0 \;\; \text{subject to} \; \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \qquad (2)$$

- The computational complexity for the solution of this constrained minimization is known to be NP-hard.

Compressive Sensing Prior Art

- Compressive sensing reconstruction procedure boils down to finding

$$\widehat{\boldsymbol{\alpha}} = \operatorname{argmin} \|\boldsymbol{\alpha}\|_0 \ \text{ subject to } \|\boldsymbol{\Phi\Psi\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \qquad (2)$$

- The computational complexity for the solution of this constrained minimization is known to be NP-hard.
- Compressive sensing idea gets attractive when this prohibitive optimization based reconstruction procedure gets replaced with a much lesser demanding ℓ_1-norm based optimization.

Compressive Sensing Prior Art

- Compressive sensing reconstruction procedure boils down to finding

$$\widehat{\boldsymbol{\alpha}} = \operatorname{argmin} \|\boldsymbol{\alpha}\|_0 \ \text{ subject to } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \tag{2}$$

- The computational complexity for the solution of this constrained minimization is known to be NP-hard.
- Compressive sensing idea gets attractive when this prohibitive optimization based reconstruction procedure gets replaced with a much lesser demanding ℓ_1-norm based optimization.

$$\widehat{\boldsymbol{\alpha}} = \operatorname{argmin} \|\boldsymbol{\alpha}\|_1 \ \text{ subject to } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \tag{3}$$

Compressive Sensing Prior Art

- Compressive sensing reconstruction procedure boils down to finding

$$\widehat{\boldsymbol{\alpha}} = \text{argmin } \|\boldsymbol{\alpha}\|_0 \text{ subject to } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \qquad (2)$$

- The computational complexity for the solution of this constrained minimization is known to be NP-hard.
- Compressive sensing idea gets attractive when this prohibitive optimization based reconstruction procedure gets replaced with a much lesser demanding ℓ_1-norm based optimization.

$$\widehat{\boldsymbol{\alpha}} = \text{argmin } \|\boldsymbol{\alpha}\|_1 \text{ subject to } \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\alpha} - \boldsymbol{y}\|_2 \leqslant \epsilon \qquad (3)$$

- The ℓ_1-norm based optimization criterion leads to well studied algorithms such as Basis Pursuit.

Multirate Observations meet Compressive Sensing

Multirate Observations meet Compressive Sensing

- Multirate data acquisition scheme can be recast as a CS type data acquisition protocol.

Multirate Observations meet Compressive Sensing

- Multirate data acquisition scheme can be recast as a CS type data acquisition protocol.
- The multirate and multichannel filtering and downsampling steps can be conjoined in a single linear projection operator.

Multirate Observations meet Compressive Sensing

- Multirate data acquisition scheme can be recast as a CS type data acquisition protocol.
- The multirate and multichannel filtering and downsampling steps can be conjoined in a single linear projection operator.
- We assume FIR filters with impulse responses $\boldsymbol{h}_i$ of length H in the individual channels.

Multirate Observations meet Compressive Sensing

- Multirate data acquisition scheme can be recast as a CS type data acquisition protocol.
- The multirate and multichannel filtering and downsampling steps can be conjoined in a single linear projection operator.
- We assume FIR filters with impulse responses $\boldsymbol{h}_i$ of length H in the individual channels.
- Hence, the observation vectors can be written as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{D}_{\downarrow}^{i}\left(\boldsymbol{h}_i * \boldsymbol{x}\right) \\ &= \boldsymbol{D}_{\downarrow}^{i}\boldsymbol{H}_i\boldsymbol{x} \\ &= \boldsymbol{\Phi}_i\boldsymbol{x} \end{aligned} \tag{4}$$

Multirate Observations meet Compressive Sensing

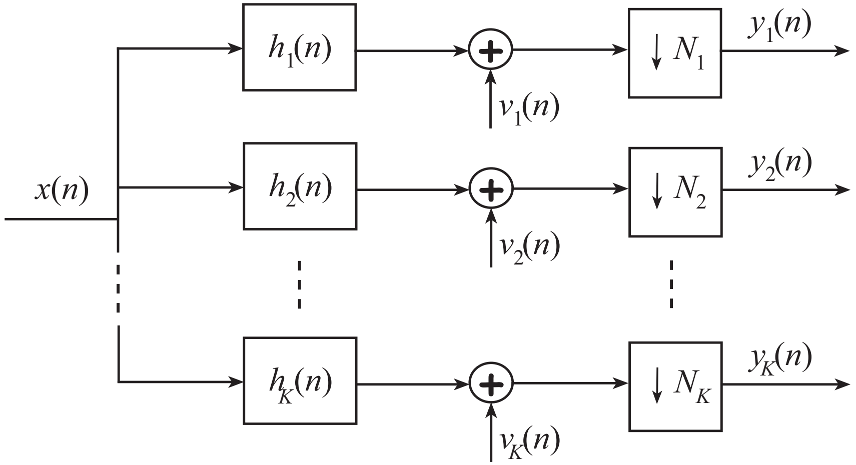

Figure 2: Multirate multichannel signal observation mechanism.

Multirate Observations meet Compressive Sensing

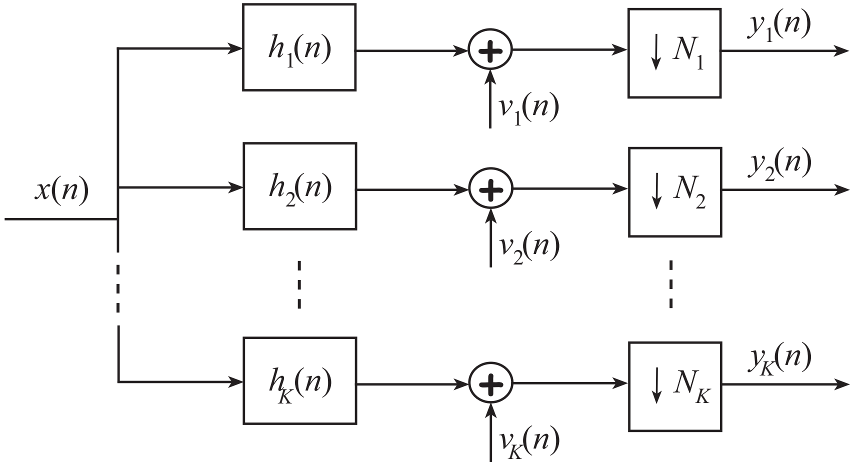

Figure 2: Multirate multichannel signal observation mechanism.

- $D^i_{\downarrow}$ is the downsampling matrix with the downsampling ratio N_i.

Multirate Observations meet Compressive Sensing

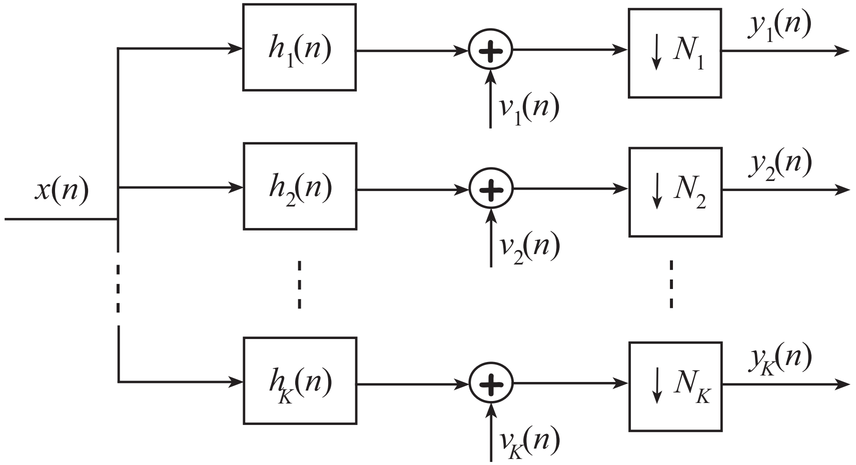

Figure 2: Multirate multichannel signal observation mechanism.

- $\boldsymbol{D}_{\downarrow}^{i}$ is the downsampling matrix with the downsampling ratio N_i.
- $\boldsymbol{H}_i$ is the convolution matrix corresponding to the FIR filter with the impulse response $\boldsymbol{h}_i$ in the i^{th} channel.

Multirate Observations meet Compressive Sensing

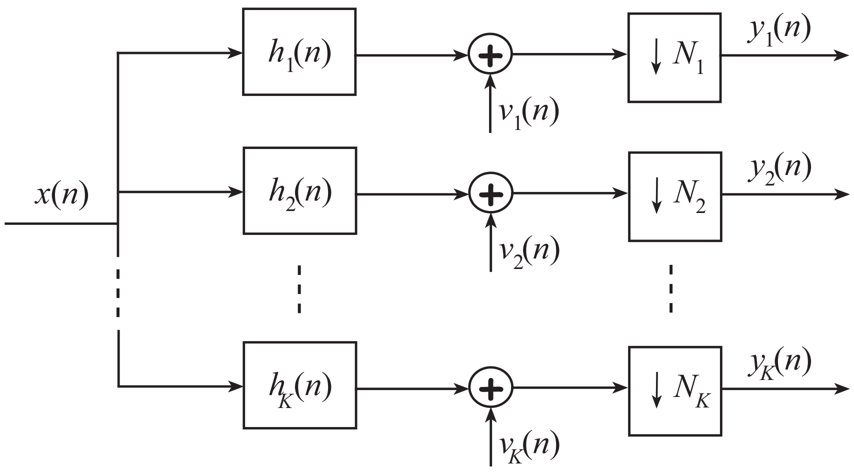

Figure 2: Multirate multichannel signal observation mechanism.

- $\boldsymbol{D}_{\downarrow}^{i}$ is the downsampling matrix with the downsampling ratio N_i.
- $\boldsymbol{H}_i$ is the convolution matrix corresponding to the FIR filter with the impulse response $\boldsymbol{h}_i$ in the i^{th} channel.
- $\boldsymbol{\Phi}_i = \boldsymbol{D}_{\downarrow}^{i} \boldsymbol{H}_i$ is the observation matrix for the i^{th} channel.

Multirate Observations meet Compressive Sensing

- The observations from the different channels come together to form the single big observation vector $\boldsymbol{y}$.

$$\begin{aligned}\boldsymbol{y} &= \left[\boldsymbol{y}_1^T \cdots \boldsymbol{y}_k^T\right]^T \\ &= \boldsymbol{\Phi}_{\mathrm{MR}}\boldsymbol{x}\end{aligned} \tag{5}$$

Multirate Observations meet Compressive Sensing

- The observations from the different channels come together to form the single big observation vector $\boldsymbol{y}$.

$$\begin{aligned} \boldsymbol{y} &= \left[\boldsymbol{y}_1^T \cdots \boldsymbol{y}_k^T\right]^T \\ &= \boldsymbol{\Phi}_{\mathrm{MR}}\boldsymbol{x} \end{aligned} \qquad (5)$$

- The overall CS projection matrix for the multirate, multichannel signal observation setting is denoted by $\boldsymbol{\Phi}_{\mathrm{MR}}$.

Multirate Observations meet Compressive Sensing

- The observations from the different channels come together to form the single big observation vector $\boldsymbol{y}$.

$$\begin{aligned} \boldsymbol{y} &= \left[\boldsymbol{y}_1^T \cdots \boldsymbol{y}_k^T\right]^T \\ &= \boldsymbol{\Phi}_{\mathrm{MR}}\boldsymbol{x} \end{aligned} \tag{5}$$

- The overall CS projection matrix for the multirate, multichannel signal observation setting is denoted by $\boldsymbol{\Phi}_{\mathrm{MR}}$.
- $\boldsymbol{\Phi}_{\mathrm{MR}}$ is generated by concatenating all the observation matrices $\boldsymbol{\Phi}_i$ corresponding to the individual channels together.

$$\boldsymbol{\Phi}_{\mathrm{MR}} = \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \vdots \\ \boldsymbol{\Phi}_K \end{bmatrix} \tag{6}$$

Multirate Observations meet Compressive Sensing

- Assume that the observed signal $\boldsymbol{x}$ has a sparse representation in some basis $\boldsymbol{\Psi}$

$$\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{\alpha}$$

Multirate Observations meet Compressive Sensing

- Assume that the observed signal $\boldsymbol{x}$ has a sparse representation in some basis $\boldsymbol{\Psi}$

$$\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{\alpha}$$

- CS multirate data acquisition can be written as

$$\boldsymbol{y} = \boldsymbol{\Theta}_{\mathrm{MR}}\boldsymbol{\alpha} \tag{7}$$

Multirate Observations meet Compressive Sensing

- Assume that the observed signal $\boldsymbol{x}$ has a sparse representation in some basis $\boldsymbol{\Psi}$
$$\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{\alpha}$$
- CS multirate data acquisition can be written as
$$\boldsymbol{y} = \boldsymbol{\Theta}_{\mathrm{MR}}\boldsymbol{\alpha} \tag{7}$$
- $\boldsymbol{\Theta}_{\mathrm{MR}} = \boldsymbol{\Phi}_{\mathrm{MR}}\boldsymbol{\Psi}$ is the sensing matrix starting from the sparse domain.

Numerical Results

Numerical Results

- Numerical results for CS based reconstruction for multirate signal observation problem are presented.

Numerical Results

- Numerical results for CS based reconstruction for multirate signal observation problem are presented.
- The experiments study the probability of exact reconstruction for the novel CS based approach to signal reconstruction from multichannel multirate observations.

Numerical Results

- Numerical results for CS based reconstruction for multirate signal observation problem are presented.
- The experiments study the probability of exact reconstruction for the novel CS based approach to signal reconstruction from multichannel multirate observations.
- In the reconstruction from the CS measurements step, we utilize the ℓ_1-Magic toolbox as developed by Candès and Romberg.

Numerical Results

- In this work we present results for signals sparse in the Discrete Cosine Transform (DCT) domain.

Numerical Results

- In this work we present results for signals sparse in the Discrete Cosine Transform (DCT) domain.
- In the experiments, signal length is fixed at $n = 128$ and the sparsity is fixed at $S = 10$.

Numerical Results

- In this work we present results for signals sparse in the Discrete Cosine Transform (DCT) domain.
- In the experiments, signal length is fixed at $n = 128$ and the sparsity is fixed at $S = 10$.
- H filter impulse response tabs for each of the distinct K channels are generated randomly from an $\mathcal{N}(0, 1)$ distribution.

Numerical Results

- In this work we present results for signals sparse in the Discrete Cosine Transform (DCT) domain.
- In the experiments, signal length is fixed at $n = 128$ and the sparsity is fixed at $S = 10$.
- H filter impulse response tabs for each of the distinct K channels are generated randomly from an $\mathcal{N}(0, 1)$ distribution.
- We consider in the experiments the scenario with two and three multirate sampling channels.

Numerical Results

- In this work we present results for signals sparse in the Discrete Cosine Transform (DCT) domain.
- In the experiments, signal length is fixed at $n = 128$ and the sparsity is fixed at $S = 10$.
- H filter impulse response tabs for each of the distinct K channels are generated randomly from an $\mathcal{N}(0, 1)$ distribution.
- We consider in the experiments the scenario with two and three multirate sampling channels.
- The subsampling rates in the different channels are equivalent.

Numerical Results

- Consider the case for two sensing channels with $N_1 = N_2$.

Numerical Results

- Consider the case for two sensing channels with $N_1 = N_2$.
- The filter lengths are chosen as $H = 4, 8, 16, 32, 64$.

Numerical Results

- Consider the case for two sensing channels with $N_1 = N_2$.
- The filter lengths are chosen as $H = 4, 8, 16, 32, 64$.
- We also present results for a fully random i.i.d sensing matrix with entries chosen from a normal distribution.

Numerical Results

- Consider the case for two sensing channels with $N_1 = N_2$.
- The filter lengths are chosen as $H = 4, 8, 16, 32, 64$.
- We also present results for a fully random i.i.d sensing matrix with entries chosen from a normal distribution.

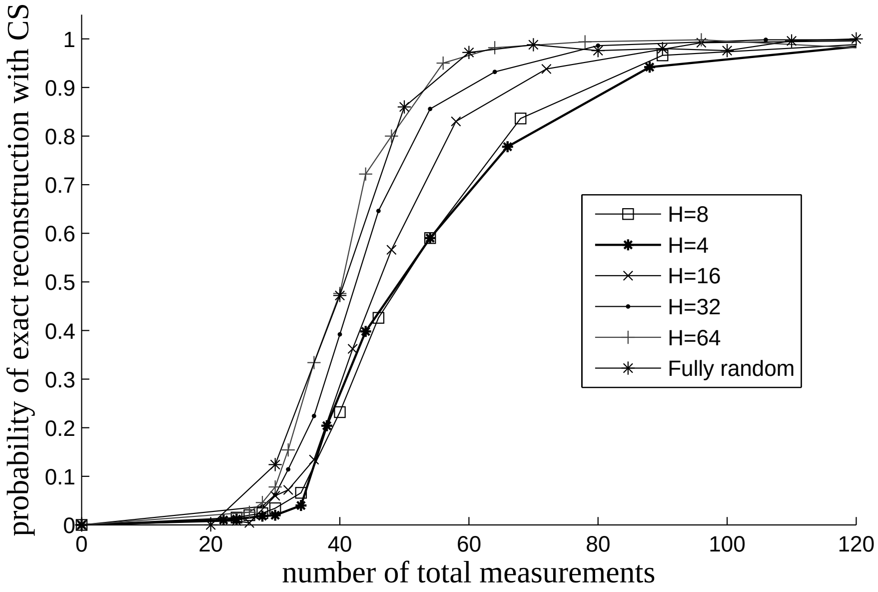

Figure 3: Probability of exact reconstruction versus the length of the total observation vector $\boldsymbol{y}$ for $N_1 = N_2$ with differing filter lengths.

Numerical Results

- Secondly we consider the case with $N_1 = N_2 = N_3$.

Numerical Results

- Secondly we consider the case with $N_1 = N_2 = N_3$.
- The filter lengths are chosen as $H = 4, 8, 16, 32, 64$.

Numerical Results

- Secondly we consider the case with $N_1 = N_2 = N_3$.
- The filter lengths are chosen as $H = 4, 8, 16, 32, 64$.

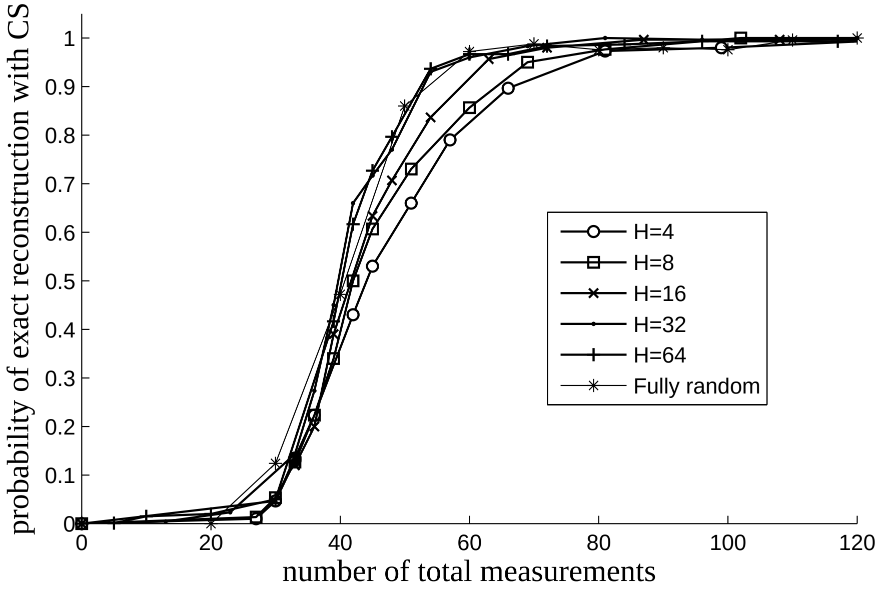

Figure 4: Probability of exact reconstruction versus the length of the total observation vector $\boldsymbol{y}$ for $N_1 = N_2 = N_3$ with differing filter lengths.

Numerical Results

- We observe that longer filter lengths translate into better reconstruction performance.

Numerical Results

- We observe that longer filter lengths translate into better reconstruction performance.
- In these figures we also present results for a fully random i.i.d sensing matrix with entries chosen from a normal distribution.

Numerical Results

- We observe that longer filter lengths translate into better reconstruction performance.
- In these figures we also present results for a fully random i.i.d sensing matrix with entries chosen from a normal distribution.
- We realized CS measurements and reconstruction using this fully random matrix for comparison purposes.

Numerical Results

- The results for the fully random matrix, two channel multirate measurements with $N_1 = N_2$ and $H = 64$, and three channel multirate measurements with $N_1 = N_2 = N_3$ and $H = 64$ are represented below.

Numerical Results

- The results for the fully random matrix, two channel multirate measurements with $N_1 = N_2$ and $H = 64$, and three channel multirate measurements with $N_1 = N_2 = N_3$ and $H = 64$ are represented below.

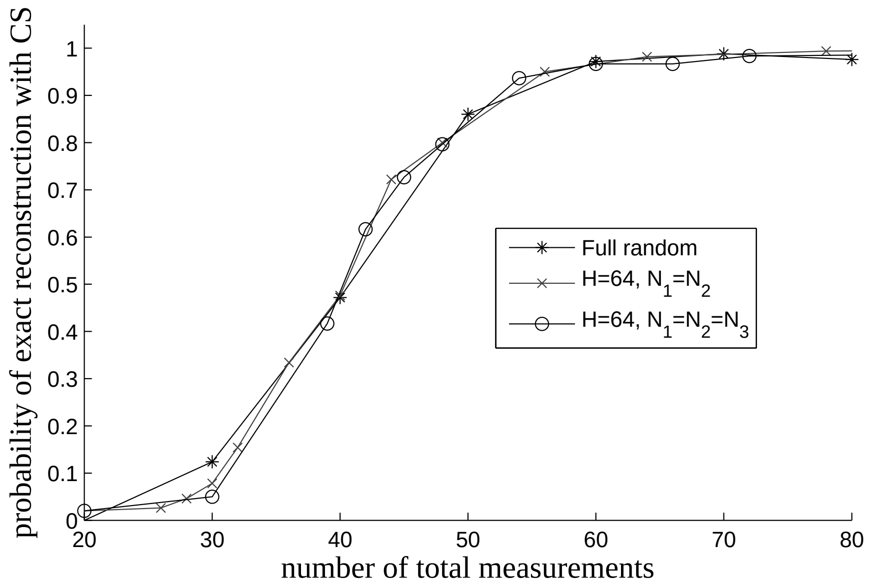

Figure 5: Probability of exact reconstruction for fully random sensing matrix, $N_1 = N_2$ multirate system with $H = 64$ and $N_1 = N_2 = N_3$ multirate system with $H = 64$.

Numerical Results

- For sufficiently long filter length H, the multichannel multirate sampling schemes in the CS setting work with a performance comparable to fully random CS sensing matrices.

Numerical Results

- For sufficiently long filter length H, the multichannel multirate sampling schemes in the CS setting work with a performance comparable to fully random CS sensing matrices.
- Multirate multichannel data acquisition system presents a viable sensing mechanism for CS.

Conclusions

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.
- Numerical results suggest the suitability of this type of data acquisition for compressible signals sparse in the DCT domain.

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.
- Numerical results suggest the suitability of this type of data acquisition for compressible signals sparse in the DCT domain.
- There is a plethora of subjects remaining for future work.

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.
- Numerical results suggest the suitability of this type of data acquisition for compressible signals sparse in the DCT domain.
- There is a plethora of subjects remaining for future work.
- Work on signals sparse in different transform domains

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.
- Numerical results suggest the suitability of this type of data acquisition for compressible signals sparse in the DCT domain.
- There is a plethora of subjects remaining for future work.
- Work on signals sparse in different transform domains
- Establishing RIP results for the CS matrices occurring in this acquisition setup

Conclusions

- We have shown that multichannel multirate signal acquisition might become a viable CS sensing mechanism for compressible signals.
- Numerical results suggest the suitability of this type of data acquisition for compressible signals sparse in the DCT domain.
- There is a plethora of subjects remaining for future work.
- Work on signals sparse in different transform domains
- Establishing RIP results for the CS matrices occurring in this acquisition setup
- Evaluating the effect of unequal subsampling rates in the different channels on the reconstruction performance

Thanks

Thanks

Thanks for listening.