NONLINEAR SYSTEM IDENTIFICATION USING DETERMINISTIC MULTILEVEL SEQUENCES

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MAIN HEADINGS

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PURPOSE

- Nonlinear system identification is important due to the shortcoming of linear models when applied to inherently nonlinear problems which are abundant in real life applications.
- The truncated (or “doubly finite”) Volterra series representation constitutes an appealing nonlinear system model, since the output is linearly dependent on the kernel parameters, hence making the identification process mathematically tractable.
- This paper proposes a novel partitioning of the Volterra kernels, resulting in simple closed form solutions when deterministic multilevel input sequences are used.

MULTIVARIATE KERNEL VECTOR REPRESENTATION

- **The usual Volterra filter representation:**

$$y(n) = \mathcal{N}[x(n)]$$

$$= \sum_{k=1}^{M} \sum_{i_1=0}^{N} \sum_{i_2=i_1}^{N} \cdots \sum_{i_k=i_{k-1}}^{N} b_k\left(i_1, i_2, \ldots, i_k\right) x(n-i_1) x(n-i_2) \cdots x(n-i_k)$$

Here, M is the order and N is the memory length of the Volterra filter and $b_k\left(i_1, i_2, \ldots, i_k\right)$ is the triangular Volterra kernel of degree k.

- The kernels $b_k\,(i_1, i_2, \ldots, i_k)$ are grouped together according to the order k of the nonlinear input term $x(n-i_1)x(n-i_2)\cdots x(n-i_k)$. We propose a different grouping for the kernels. The output $y(n)$ can be rewritten as the sum of the outputs of M different multivariate cross-term nonlinear subsystems, $\mathcal{H}^{(\ell)}$.

- **The proposed Volterra filter representation:**

$$y(n) = \mathcal{N}[x(n)] = \sum_{\ell=1}^{M} y^{(\ell)}(n) = \sum_{\ell=1}^{M} \mathcal{H}^{(\ell)}[x(n)]$$

$$\mathcal{H}^{(\ell)}[x(n)] \;=$$

$$\begin{cases} \displaystyle\sum_{i=0}^{N} \mathbf{h}^{(1)^T}(i)\,\mathbf{x}_h^{(1)}(n-i) & \ell = 1 \\ \displaystyle\sum_{q_1=1}^{Q_1} \cdots \sum_{q_{\ell-1}=1}^{Q_{\ell-1}} \sum_{i=0}^{I_{\bar{q}_{\ell-1}}} \mathbf{h}^{(\ell)^T}(q_1, \ldots, q_{\ell-1}; i)\,\mathbf{x}_h^{(\ell)}(q_1, \ldots, q_{\ell-1}; n-i) & 2 \leqslant \ell \leqslant M \end{cases}$$

- In this representation, the symbol $\mathcal{H}^{(\ell)}[\cdot]$, which represents ℓ summations, is called as an ℓ-D cross-term Volterra operator and $\mathbf{h}^{(\ell)}(q_1, \ldots, q_{\ell-1}; i)$ is called as an ℓ-D kernel vector.

- The ℓ-D input vector can be expressed in the following form::

$$\mathbf{x}_h^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) = \begin{bmatrix} x_{h,\ell}^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) \\ \mathbf{x}_{h,\ell+1}^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) \\ \vdots \\ \mathbf{x}_{h,M}^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) \end{bmatrix}$$

Here, the subinput vectors $\mathbf{x}_{h,k}^{(\ell)}(q_1, \ldots, q_{\ell-1}; n)$ consist of all possible inputs of degree k,

$$x_{h,k}^{(p_1,\cdots,p_\ell)}(q_1, \ldots, q_{\ell-1}; n) \hat{=} x^{p_1}(n)\, x^{p_2}(n-q_1) \cdots x^{p_\ell}(n-q_1-\cdots-q_{\ell-1})$$

- The corresponding ℓ-D kernel vector $\mathbf{h}^{(\ell)}(q_1, \ldots, q_{\ell-1}; i)$ can be rewritten in terms of subkernels. There exists an equivalent triangular kernel $b_k(i_1, i_2, \ldots, i_k)$ for each component of the subkernel vector $\mathbf{h}_k^{(\ell)}(q_1, \ldots, q_{\ell-1}; i)$.
- We introduced the concept of delay-wise dimensionality and cross-term subsystem to replace the multiplicational dimensionality of the regular Volterra kernels. This novel grouping enables us to devise an exact closed form algorithm for identifying the Volterra kernels using deterministic multilevel sequences.

IDENTIFICATION OF THE KERNEL VECTORS USING MULTILEVEL INPUT SIGNALS

- In this section, we derive an efficient algorithm to identify the kernel vectors $\mathbf{h}^{(\ell)}(q_1, \ldots, q_{\ell-1}; i)$ by using multilevel input sequences with ℓ distinct impulses.

- **Identification of the 1-D Kernel Vectors:**
Multilevel single impulses, $x^{(1)}(m_1; n) = a_{m_1}\ \delta(n)$, for $m_1 = 1, 2, \ldots, \binom{M}{1}$ can be used to obtain the 1-D kernel vectors. Using the cross-term representation, it is trivial to prove that the higher dimensional outputs are zero for these multilevel single impulses, i.e., $y^{(\ell)}(n) = 0$ for $\ell > 1$. Hence,

$$y^{(1)}(m_1; n) = \mathcal{N}\Big[x^{(1)}(m_1; n)\Big] = \sum_{i=0}^{N} \mathbf{h}^{(1)^T}(i)\mathbf{u}_h^{(1)}(m_1; n-i)$$

$$\mathbf{u}_h^{(1)}(m_1; n) = \big[x(m_1; n)\ x^2(m_1; n)\ \cdots\ x^M(m_1; n)\big]^T$$

Now we can write all $\binom{M}{1}$ ensemble outputs in the matrix form as follows::

$$\mathbf{y}_e^{(1)}(n) = \mathcal{H}^{(1)}\left[\mathbf{x}_e^{(1)}(n)\right] = \mathbf{U}_e^{(1)}\,\mathbf{h}^{(1)}(n)$$

Here, $\mathbf{x}_e^{(1)}(n), \mathbf{y}_e^{(1)}(n)$ and $\mathbf{U}_e^{(1)}(n)$ denote the ensemble input, ensemble output vectors and the ensemble input matrix, respectively.

Therefore, provided the inverse of the $M \times M$ matrix $\mathbf{U}_e^{(1)}$ exists, all the 1-D kernel vectors can be obtained as:

$$\mathbf{h}^{(1)}(n) = \left[\mathbf{U}_e^{(1)}\right]^{-1} \mathbf{y}_e^{(1)}(n)$$

This result shows that all 1-D kernels with one cross-term can be determined by using only the inverse of an M ensemble matrix times the ensemble output matrix. Note that the linear FIR filter identification via the impulse response is covered by this method as the special case $M = 1$.

- **Identification of the ℓ-D Kernel Vectors:** The ℓ-D input ensemble vector can be written as:

$$\mathbf{x}_e^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) = \sum_{i=1}^{\ell} \mathbf{T}_{\ell,i}^{(M)} \mathbf{x}_e^{(1)}(n - n_i^{(\ell)})$$

Let $\mathbf{v}_e^{(m,k)}(q_1, \ldots, q_{m-1}; n)$ denote the ensemble output of the k-D subsystem $\mathcal{H}^{(k)}$ from the m-D input ensemble $\mathbf{x}_e^{(m)}(q_1, \ldots, q_{m-1}; n)$.

- Using these definitions, the response of the nonlinear system to the ensemble input in $\mathbf{x}_e^{(\ell)}(q_1, \ldots, q_{\ell-1}; n)$ can be written in terms of the outputs of the subsystems.

$$\mathbf{y}_e^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) = \sum_{k=1}^{\ell} \mathbf{v}_e^{(\ell,k)}(q_1, \ldots, q_{\ell-1}; n)$$

- The subsystem outputs $\mathbf{v}_e^{(\ell,k)}(q_1, \ldots, q_{\ell-1}; n)$, $k = 1, 2, \ldots, \ell - 1$ can be obtained from the previous lower dimensional subsystem outputs.

$$\mathbf{v}^{(\ell,k)}(q_1, \ldots, q_{\ell-1}; n) = \sum_{j=1}^{\binom{\ell}{k}} \mathbf{S}_{\ell 1,j}^{(M)} \mathbf{v}_e^{(k,k)}(\mathbf{q}_j^{(\ell,k)}; n - n_j^{(\ell,k)})$$

- It is possible to determine the ℓ-D Volterra kernel vectors by:

$$\mathbf{h}^{(\ell)}(q_1, \ldots, q_{\ell-1}; n - \bar{q}_{\ell-1}) = \left[\mathbf{U}_e^{(\ell)}\right]^{-1} \mathbf{v}_e^{(\ell,\ell)}(q_1, \ldots, q_{\ell-1}; n)$$

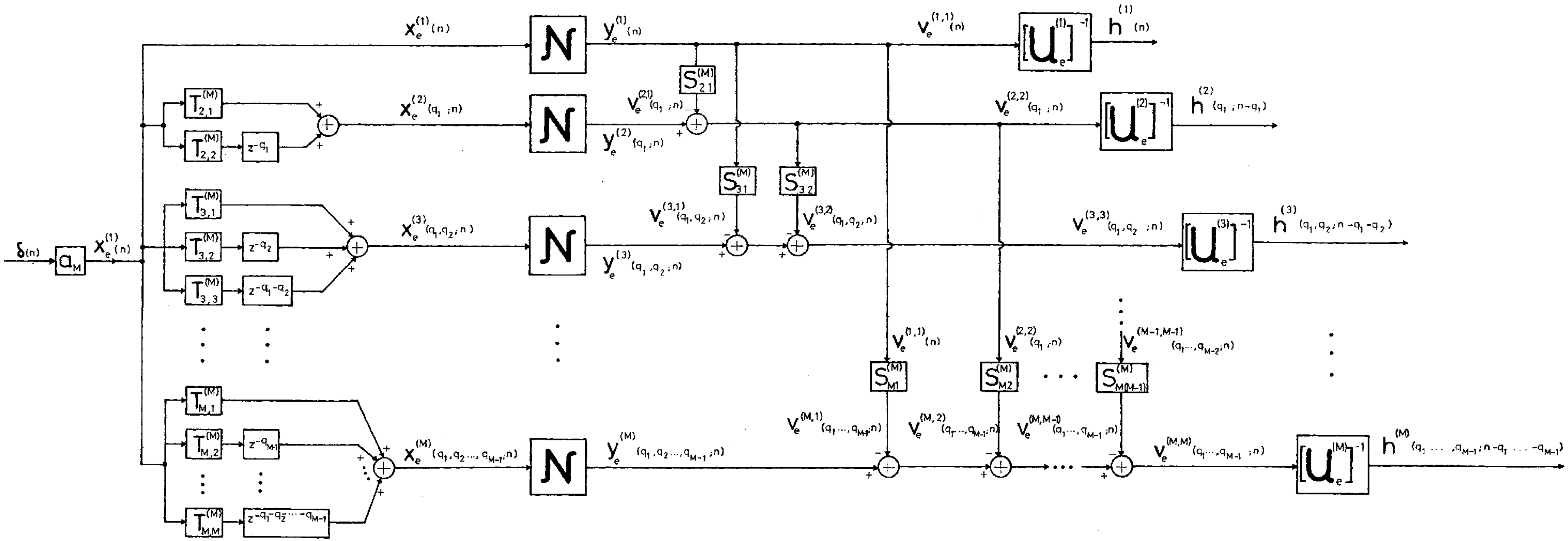
Here,

$$\mathbf{v}_e^{(\ell,\ell)}(q_1, \ldots, q_{\ell-1}; n) = \mathbf{y}_e^{(\ell)}(q_1, \ldots, q_{\ell-1}; n) - \sum_{j=1}^{\binom{\ell}{1}} \mathbf{S}_{\ell 1,j}^{(M)} \mathbf{v}_e^{(1,1)}(n - n_j^{(\ell,1)})$$

$$- \sum_{k=2}^{\ell-1} \sum_{j=1}^{\binom{\ell}{k}} \mathbf{S}_{\ell k,j}^{(M)} \mathbf{v}_e^{(k,k)}(q_{j,i}^{(\ell,k)}, \ldots, q_{j,k-1}^{(\ell,k)}; n - n_j^{(\ell,k)})$$

- $\mathbf{U}_e^{(\ell)}$ is an $\binom{M}{\ell} \times \binom{M}{\ell}$ input ensemble matrix composed of terms in the form of $\left(a_{i_1}^{p_1} a_{i_2}^{p_2} \cdots a_{i_\ell}^{p_\ell}\right)$.
- Fig. 1 depicts the identification of the Volterra kernels of orders one through M using the proposed algorithm.

Figure 1. Proposed Volterra kernel identification method using multilevel deterministic sequences as inputs.



SIMULATIONS

- Pseudorandom multilevel sequences (PRMS) which can be chosen to be persistently exciting (PE) for any finite order Volterra system were previously considered for nonlinear system identification. However, the condition on the order of the PRMS to ensure PE is sufficient but not necessary for the regular Volterra filter. Hence, PRMS includes redundant input sequences when the system under consideration is a regular Volterra filter.

- We simulate a second order Volterra filter with $N = 2$. The average input power is unity and independent GWN of power 0.1 is added to the system output to represent observation noise.
- In Table 1, the input sequence lengths, averaged squared error between the estimated and true kernels over 1000 independent trials and the number of floating point operations required are given. Our algorithm uses much less number of operations and gives better results than the PRMS method .

TABLE 1

COMPARISON WITH PRMS METHOD

PRMS			multilevel deterministic sequence		
length	error	of ops.	length	error	of ops.
27	7.80×10^{-1}	1.22×10^{3}	15	2.25×10^{-1}	0.12×10^{3}
64	9.93×10^{-2}	1.64×10^{3}	60	5.62×10^{-2}	0.29×10^{3}
125	2.89×10^{-2}	2.24×10^{3}	120	2.85×10^{-2}	0.53×10^{3}
343	6.18×10^{-3}	4.12×10^{3}	330	1.00×10^{-1}	1.34×10^{3}

- In the PRMS method, to get an exact least squares solution in the absence of observation noise, a full period of the PRMS, i.e., a sequence of length $(M+1)^{N+1}$ has to be used. For example, for $(M=3)$ and $(N=11)$, to get the exact solution a data record of length 4^{12} is required. However, our algorithm completely eliminates input combinations which are not required for the identification of the regular Volterra system and a sequence of length less than $(2N+1)\binom{N+M}{M-1}=2093$ is adequate for the exact least squares solution.
- This is a radical improvement over 4^{12} for PRMS.

CONCLUDING REMARKS

- We have developed a novel algorithm for input-output nonlinear system identification. Our algorithm is in closed-form, exact, non-iterative and a computationally efficient implementation is also presented. This algorithm can produce better parameter estimates than some existing algorithms. It might facilitate better Volterra kernel estimates for short input sequences. Hence, this identification algorithm might be used in the implementation of nonlinear compensators and nonlinear system inverses and equalization.

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Thanks for your kind attention.