DEPREM MÜHENDİSLİĞİNE GİRİŞ ve DEPREME DAYANIKLI YAPI TASARIMI

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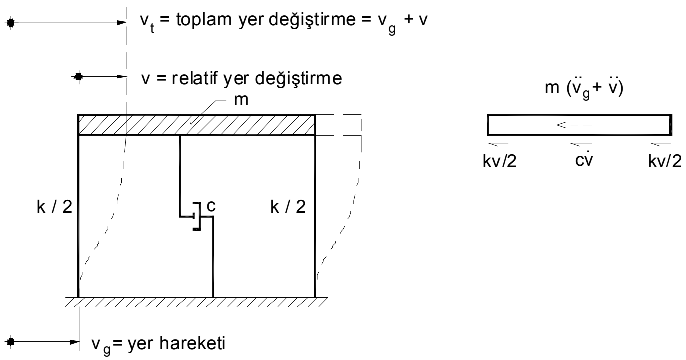
DEPREME DAYANIKLI YAPI TASARIMI

- Deprem hareketi
- Yapıların yer hareketi etkisindeki titreşimi
- Deprem etkisindeki betonarme yapı elemanlarının davranışı
- Depreme dayanıklı yapı tasarımı
- Yurdumuzdaki önemli depremler
- Yapılarda deprem sonrası hasar belirlenmesi, onarım ve güçlendirme yöntemleri
- Mevcut binaların deprem etkisindeki davranışının değerlendirilmesi

Yapıların yer hareketi etkisindeki titreşimi

Tek serbestlik dereceli sistemler



Tek serbestlik dereceli sistemler

$$m\,\ddot{v} + c\,\dot{v} + k\,v = -\,m\,\ddot{v}_g$$

$$\ddot{v} + 2\,\xi\,\omega\,\dot{v} + \omega^2\,v = -\ddot{v}_g$$

$$\omega^2 = \frac{k}{m} \qquad c_{cr} = 2\,m\,\omega \qquad \xi = \frac{c}{2\,m\,\omega} = \frac{c}{c_{cr}}$$

Tek serbestlik dereceli sistemler

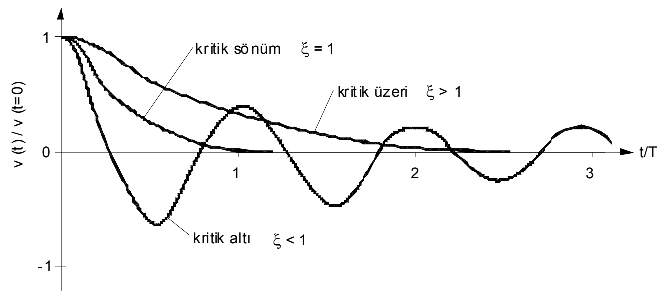
- **Hareketin başlangıç şartları**

$$v_o = v(t=0) \qquad \dot{v}_o = \dot{v}(t=0)$$

$$v(t) = e^{-\xi\omega t}\left[v_o \cos\omega_D t + \frac{\dot{v}_o + \xi\omega v_o}{\omega_D} \sin\omega_D t \right]$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad T_D = \frac{2\pi}{\omega_D} = \frac{T}{\sqrt{1-\xi^2}} \qquad \omega_D = \omega\sqrt{1-\xi^2}$$

Tek serbestlik dereceli sistemler Yerdeğiştirme



Tek serbestlik dereceli sistemlerde sönüm oranı

$$\frac{v(t)}{v(t+T_D)} = \exp(\xi\omega T_D) = \exp\left[\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right]$$

$$\delta = \ln\frac{v(t)}{v(t+T_D)} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \approx 2\pi\xi \qquad \sqrt{1-\xi^2} \approx 1$$

$$\delta = \frac{1}{m}\ln\frac{v(t)}{v(t+mT_D)} \approx 2\pi\xi$$

Yer hareketi altındaki tek serbestlik dereceli sistemler

- Yer hareketi ivme kaydı $\ddot{v}_g(t)$
- Yerdeğiştirme (Duhammel integrali)

$$v(t) = -\frac{1}{\omega_D}\int_0^t \ddot{v}_g(\tau)\, e^{-\xi\omega(t-\tau)} \sin\omega_D(t-\tau)\, d\tau$$

Çok serbestlik dereceli sistem

$\mathbf{m}\,\ddot{\mathbf{v}} + \mathbf{c}\,\dot{\mathbf{v}} + \mathbf{k}\,\mathbf{v} = -\mathbf{m}\,\mathbf{1}\,\ddot{v}_g$ $\qquad \mathbf{1} = [1 \;\; 1 \;\; .... \;\; 1]^T$

- Kütle matrisi $\qquad \mathbf{m} = [m_{ij}]$
- Sönüm matrisi $\qquad \mathbf{c} = [c_{ij}]$
- Rijitlik matrisi $\qquad \mathbf{k} = [k_{ij}]$
- Esneklik matrisi $\qquad \mathbf{d} = [d_{ij}]$
- Yerdeğiştirme vektörü $\qquad \mathbf{v} = [v_i]$
- Yer hareketi $\qquad v_g(t)$

Sönümsüz sistemin serbest titreşim

- Hareket denklemi $\qquad \mathbf{m}\,\ddot{\mathbf{v}} + \mathbf{k}\,\mathbf{v} = \mathbf{0}$
- Çözüm kabülü $\qquad \mathbf{v}(t) = \mathbf{v}_o \sin \omega t$

$$(\mathbf{k} - \omega^2 \mathbf{m})\,\mathbf{v_o} = \mathbf{0}$$

- Frekans denklemi $\qquad |\mathbf{k} - \omega^2 \mathbf{m}| = 0$
- Katsayılar determinantı
- Frekanslar $\qquad \omega_1,\; \omega_2, \dots \omega_n$
- Öz vektörler $\qquad (\mathbf{k} - \omega_i^2 \mathbf{m})\,\boldsymbol{\phi}_i = \mathbf{0}$
- Mod şekilleri

Çok serbestlik dereceli sistem

- Mod şekillerinin dikliği
- Kütle matrisine göre $\qquad \boldsymbol{\phi}_i^T\,\mathbf{m}\,\boldsymbol{\phi}_j = 0$
- Rijitlik matrisine göre $\qquad \boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_j = 0$
- Sönüm matrisine göre (kabul) $\qquad \boldsymbol{\phi}_i^T\,\mathbf{c}\,\boldsymbol{\phi}_j = 0$

$$i \neq j$$

Modların birleştirilmesi

$$\mathbf{v}(t) = \sum_{i=1}^{N} \boldsymbol{\phi}_i\, Y_i(t) \qquad Y_j = \boldsymbol{\phi}_j^T\,\mathbf{m}\,\mathbf{v} / M_j$$

$Y_j = \boldsymbol{\phi}_j^T\,\mathbf{m}\,\mathbf{v} / M_j$	Genelleştirilmiş koordinat
$M_j = \boldsymbol{\phi}_j^T\,\mathbf{m}\,\boldsymbol{\phi}_j$	Genelleştirilmiş kütle
$C_j = \boldsymbol{\phi}_j^T\,\mathbf{c}\,\boldsymbol{\phi}_j = 2\,\xi_j\,\omega_j\,M_j$	Genelleştirilmiş sönüm
$K_j = \boldsymbol{\phi}_j^T\,\mathbf{k}\,\boldsymbol{\phi}_j = \omega_j^2\,M_j$	Genelleştirilmiş rijitlik

Modların birleştirilmesi

- **Çözüm için kabul**

$$\mathbf{v}(t) = \sum_{i=1}^{N} \mathbf{v}_i = \sum_{i=1}^{N} \boldsymbol{\phi}_i \, Y_i(t)$$

$$\ddot{Y}_j + 2\,\xi_j\,\omega_j\,\dot{Y}_j + \omega_j^2\,Y_j = -\boldsymbol{\phi}_j^T\,\mathbf{m}\,\mathbf{1}\,\ddot{v}_g\,\frac{1}{M_j}$$

Modların birleştirilmesi

$$Y_j(t) = \frac{L_j}{M_j \omega_{Dj}} V_j(t)$$

$$V_i(t) = -\int_0^t \ddot{v}_g(\tau)\, e^{-\xi_j \omega_j (t-\tau)} \sin[\omega_{Dj}(t-\tau)]\, d\tau$$

$$L_j = \boldsymbol{\phi}_j^T \mathbf{m}\,\mathbf{1}$$

Modların birleştirilmesi

$$\mathbf{f}_j(t) = \mathbf{k}\,\mathbf{v}_j(t) = \mathbf{k}\,\boldsymbol{\phi}_j\,Y_j(t) = \omega_j^2\,\mathbf{m}\,\boldsymbol{\phi}_j\,Y_j(t)$$

$$Y_{j\,\max} = \frac{L_j}{M_j\,\omega_{Dj}} S_v\,(\xi_j\;T_j)$$

$$\mathbf{f}_{j\,\max} = \omega_j^2\,\mathbf{m}\,\boldsymbol{\phi}_j\,Y_{j\,\max} = \mathbf{m}\,\boldsymbol{\phi}_j\,\frac{L_j\,\omega_j^2}{M_j\,\omega_{Dj}} S_v\,(\xi_j\;T_j)$$

Modların birleştirilmesi

$$M_j = \phi_j^T\,\mathbf{m}\,\phi_j = \begin{bmatrix}\phi_{1j} & \phi_{2j} & \phi_{nj}\end{bmatrix} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_n \end{bmatrix} \begin{bmatrix}\phi_{1j} \\ \phi_{2j} \\ \phi_{nj}\end{bmatrix} =$$

$$= m_1\,\phi_{1j}^2 + m_2\,\phi_{2j}^2 + \ldots + m_n\,\phi_{nj}^2 = \sum_{i=1}^{n} m_i\,\phi_{ij}^2$$

$$L_j = \phi_j^T\,\mathbf{m}\,\mathbf{1} = \begin{bmatrix}\phi_{1j} & \phi_{2j} & \phi_{nj}\end{bmatrix} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_n \end{bmatrix} \begin{bmatrix}1 \\ 1 \\ 1\end{bmatrix} =$$

$$= m_1\,\phi_{1j} + m_2\,\phi_{2j} + \ldots + m_n\,\phi_{nj} = \sum_{i=1}^{n} m_i\,\phi_{ij}$$

Modların birleştirilmesi

$$\mathbf{f}_{j\,\max} = \mathbf{m}\,\boldsymbol{\phi}_j \frac{L_j\, S_a\,(\xi_j\; T_j)}{M_j}$$

$$V_{b\,j} = \sum_{i=1}^{n} f_{ij\,\max} = \frac{L_j}{M_j} S_a(\xi_j, T_j) \sum_{i=1}^{n} m_i\, \phi_{i\,j} =$$

$$= \frac{L_j^2}{M_j} S_a(\xi_j, T_j) = M_j^*\, S_a(\xi_j, T_j)$$

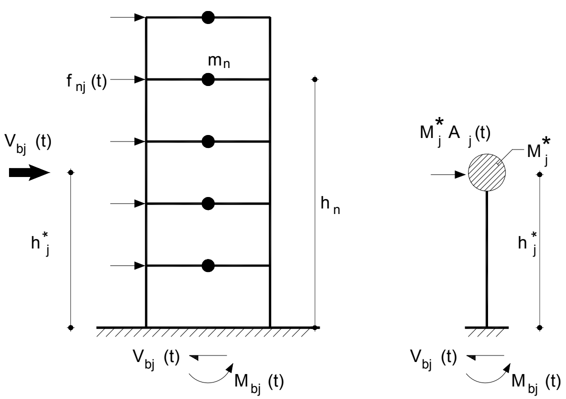
Modların birleştirilmesi

$$M_j^* = \Gamma_j\, L_j = \frac{(L_j)^2}{M_j} = \frac{\left(\sum_{i=1}^{n} m_i\, \phi_{i\,j}\right)^2}{\sum_{i=1}^{n} m_i\, \phi_{i\,j}^{\;2}}$$

$$f_{ij\,\max} = V_{b\,j} \frac{m_i\;\phi_{ij}}{\sum_{i=1}^{n} m_i\;\phi_{ij}^2}$$

$$v_{\max} = \sqrt{v_{1\max}^2 + v_{2\max}^2 + \ldots + v_{n\max}^2}$$

Modların birleştirilmesi



Sayısal integrasyon

$$\mathbf{m}\;\ddot{\mathbf{v}}\;(t) + \mathbf{c}(t)\;\dot{\mathbf{v}}\;(t) + \mathbf{k}(t)\;\mathbf{v}\;(t) = -\mathbf{m}\;\ddot{\mathbf{v}}_{\mathbf{g}}\;(t)$$

$$\mathbf{m}\;\Delta\;\ddot{\mathbf{v}}\;(t) + \mathbf{c}\;(t)\;\Delta\;\dot{\mathbf{v}}\;(t) + \mathbf{k}\;(t)\;\Delta\;\mathbf{v}\;(t) = -\mathbf{m}\;\Delta\;\ddot{\mathbf{v}}_{\mathbf{g}}(t)$$

Modların birleştirilmesi

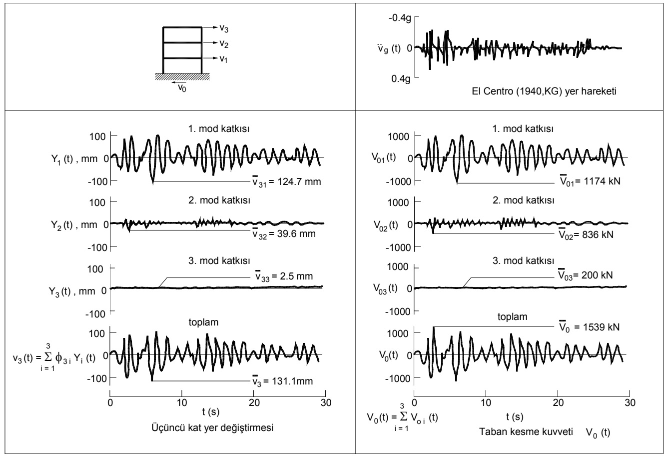
a) Mutlak değerlerin toplamı

$$r(t)_{\max} \leq \sum_{j=1}^{N} \left[r_j(t) \right]_{\max} = \left| r_{1o} \right| + \ldots + \left| r_{no} \right| + \ldots + \left| r_{No} \right|$$

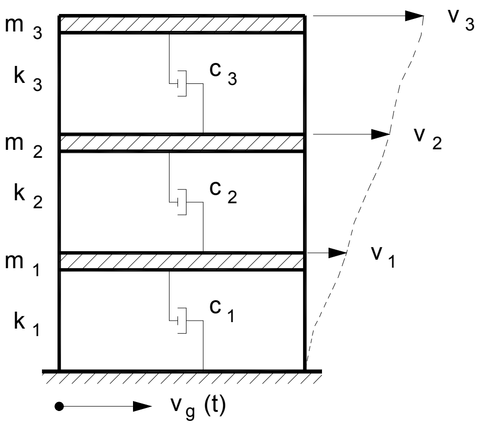
b) Karelerinin toplamının karekökü

$$r(t)_{\max} \approx \left[\sum_{j=1}^{N} \left[r_j(t) \right]_{\max}^2 \right]^{1/2} = \left[\left[r_{1o} \right]^2 + \ldots + \left[r_{no} \right]^2 + \ldots + \left[r_{No} \right]^2 \right]^{1/2}$$

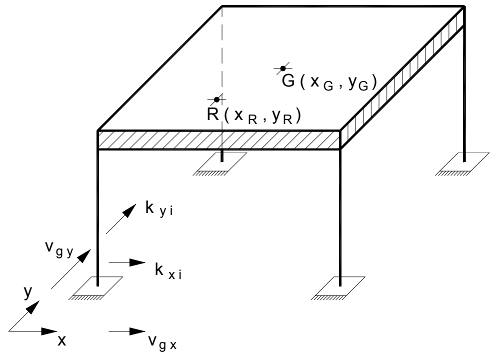
Modların birleştirilmesi



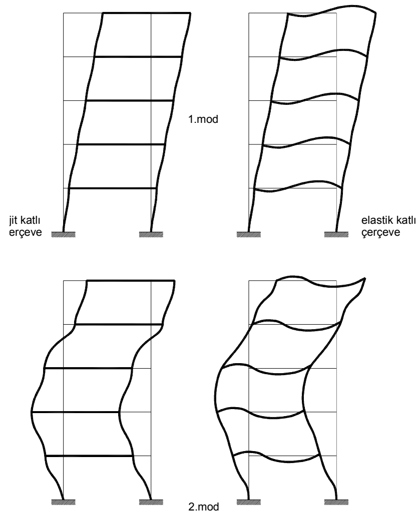
Rijit döşemeli çerçeve



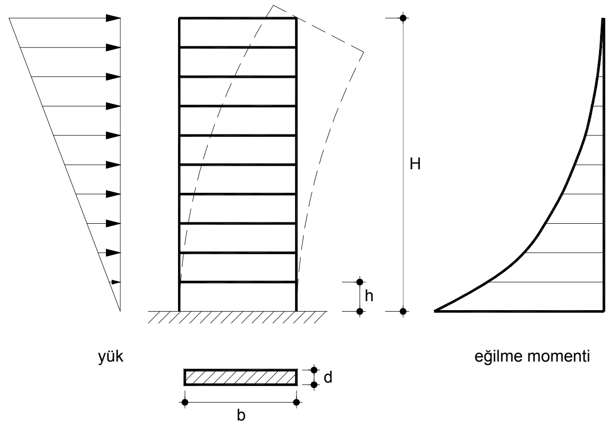
Uzay çerçeve



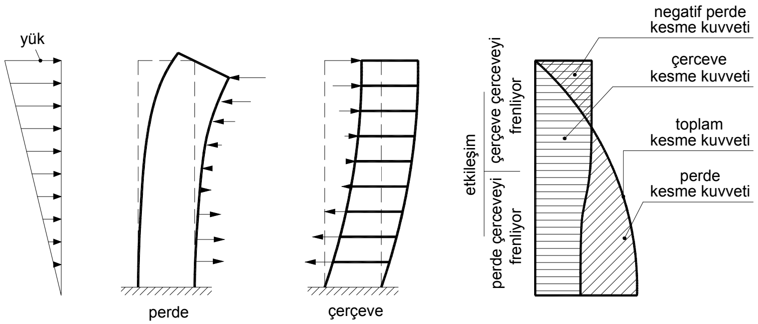
Çerçeve



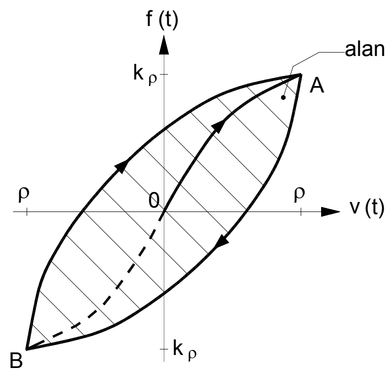
Perde davranışı



Perde çerçeve etkileşimi



Sönüm

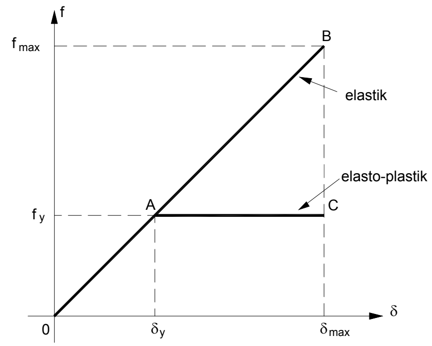


Sönüm

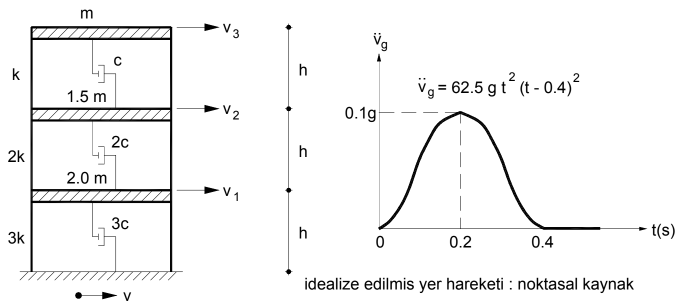
- Dış viskoz sönüm
- İç viskoz sönüm
- Coumb sönümü
- Çevrimsel sönüm
- Enerji yayılma sönümü

Süneklik

$$\mu = \delta_{max} / \delta_y$$



Örnek

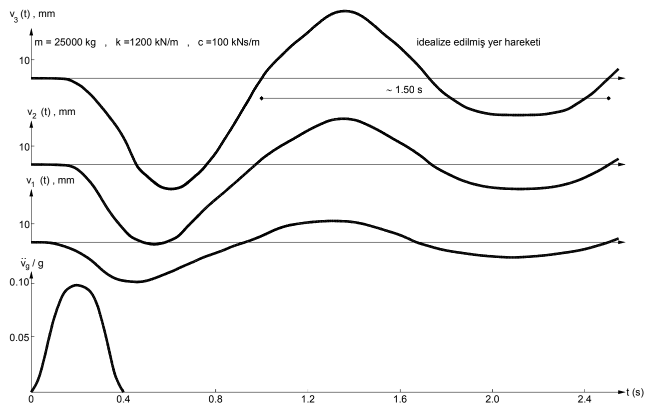


Örnek

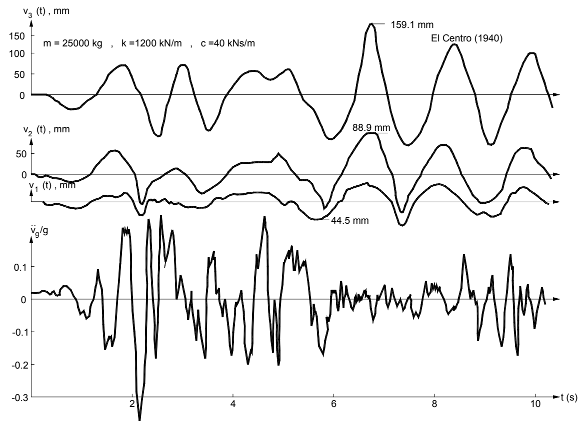
$$\mathbf{m}\ \ddot{\mathbf{v}}\ (t) + \mathbf{k}\ \mathbf{v}\ (t) = -\mathbf{m}\ \mathbf{1}\ \ddot{v}_g$$

$$\mathbf{m} = \begin{bmatrix} 2m & 0 & 0 \\ 0 & 1.5m & 0 \\ 0 & 0 & m \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 5k & -2k & 0 \\ -2k & 3k & -k \\ 0 & -k & k \end{bmatrix}$$

Örnek

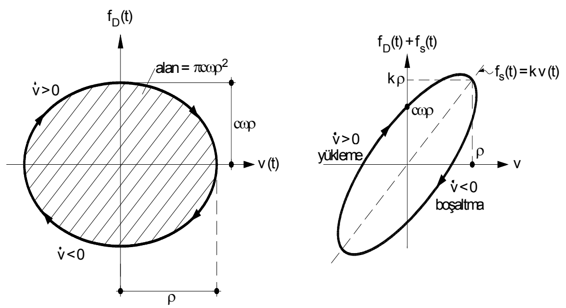


Örnek



Sönüm

$$m\,\ddot v\,(t) + f_D(t) + k\,v\,(t) = -m\,\ddot v_g\,(t)$$



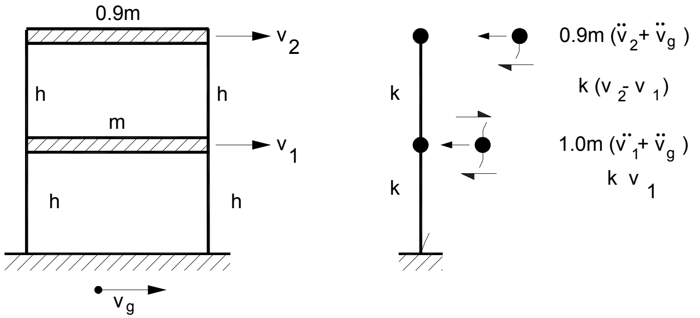
Viskoz sönüm

$$f_D(t) = c\,\dot v(t)$$

$$\Delta E = \int_o^T f_D(t)\,\dot v(t)\,dt = c\omega^2\rho^2 \int_o^{2\pi/\omega} \cos^2(\omega t - \theta)\,dt = \pi c\omega\rho^2$$

$$c = \frac{\Delta E}{\pi\omega\rho^2} \qquad \xi = \frac{\Delta E}{2k\pi\rho^2}$$

Örnek
İki katlı çerçeve



Örnek
Hareket denklemleri

$$m\,(\ddot{v}_1 + \ddot{v}_g) + k\,v_1 - k\,(v_2 - v_1) = 0$$

$$0.9m\,(\ddot{v}_2 + \ddot{v}_g) + k\,(v_2 - v_1) = 0$$

Örnek
Serbest titreşim

$$\mathbf{m}\,\ddot{\mathbf{v}}\,(t) + \mathbf{k}\;\mathbf{v}\,(t) = \mathbf{0}$$

$$\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & 0.9m \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$$

Örnek
Periyotlar

$$\left|\,\mathbf{k} - \omega^2\,\mathbf{m}\,\right| = \begin{vmatrix} 2k - m\omega^2 & -k \\ -k & k - 0.9m\omega^2 \end{vmatrix} = 0 \qquad \lambda_i = \omega_i^2\; m/k$$

$$\lambda = \frac{\omega^2\; m}{k} \qquad \begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-0.9\lambda \end{vmatrix} = 0.9\lambda^2 - 2.8\lambda + 1 = 0$$

$\lambda_1 = 0.411$ $\lambda_2 = 2.700$ $k = 20979\;kN/m$ $mg = 147\;kN$

$\omega_1 = 23.99 s^{-1}$ $\omega_2 = 61.48 s^{-1}$ $T_1 = 0.262\,s$ $T_2 = 0.102\,s$

Örnek
Titreşim mod şekilleri

$$(\mathbf{k} - \omega_i^2\ \mathbf{m})\ \phi_i = 0 \qquad \begin{bmatrix} 2-\lambda_1 & -1 \\ -1 & 1-0.9\lambda_1 \end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} 2-0.411 & -1 \\ -1 & 1-0.9\times 0.411 \end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=\begin{bmatrix} 1.589 & -1 \\ -1 & 0.629 \end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} 1.589 & -1.589\times 0.629 \\ -1 & 0.629 \end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} -1 & 0.629 \end{bmatrix}\begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=0 \qquad -\phi_{11}+0.629\,\phi_{21}=0 \qquad \begin{bmatrix}\phi_{11}\\ \phi_{21}\end{bmatrix}=\begin{bmatrix}0.629\\1.000\end{bmatrix}$$

Örnek
Titreşim mod şekilleri

$$(\mathbf{k} - \omega_i^2\ \mathbf{m})\ \phi_i = 0 \qquad \begin{bmatrix} 2-\lambda_2 & -1 \\ -1 & 1-0.9\lambda_2 \end{bmatrix}\begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} 2-2.700 & -1 \\ -1 & 1-0.9\times 2.700 \end{bmatrix}\begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=\begin{bmatrix} -0.700 & -1 \\ -1 & -1.429 \end{bmatrix}\begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} -0.700 & -0.700\times 1.429 \\ -1 & -1.429 \end{bmatrix}\begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} -1 & -1.429 \end{bmatrix}\begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=0 \qquad -\phi_{12}-1.429\,\phi_{22}=0 \qquad \begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix}=\begin{bmatrix}-1.429\\1.000\end{bmatrix}$$

Örnek
Mod şekillerinin dikliği ve genelleştirilmiş kütle

$$\boldsymbol{\phi}_1^T\ \mathbf{m}\ \boldsymbol{\phi}_2 = \begin{bmatrix}0.629 & 1.000\end{bmatrix}\begin{bmatrix} m & 0 \\ 0 & 0.9m \end{bmatrix}\begin{bmatrix}-1.429\\1.000\end{bmatrix}=0$$

$$\boldsymbol{\phi}_1^T\ \mathbf{k}\ \boldsymbol{\phi}_2 = \begin{bmatrix}0.629 & 1.000\end{bmatrix}\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix}-1.429\\1.000\end{bmatrix}=0$$

$$M_1 = \boldsymbol{\phi}_1^T\ \mathbf{m}\ \boldsymbol{\phi}_1 = \begin{bmatrix}0.629 & 1.000\end{bmatrix}\begin{bmatrix} m & 0 \\ 0 & 0.9m \end{bmatrix}\begin{bmatrix}0.629\\1.000\end{bmatrix}=1.296\,m$$

$$M_2 = \boldsymbol{\phi}_2^T\ \mathbf{m}\ \boldsymbol{\phi}_2 = \begin{bmatrix}-1.429 & 1.000\end{bmatrix}\begin{bmatrix} m & 0 \\ 0 & 0.9m \end{bmatrix}\begin{bmatrix}-1.429\\1.000\end{bmatrix}=2.942\,m$$

Örnek
Genelleştirilmiş rijitlik

$$K_1 = \boldsymbol{\phi}_1^T\ \mathbf{k}\ \boldsymbol{\phi}_1 = \begin{bmatrix}0.629 & 1.000\end{bmatrix}\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix}0.629\\1.000\end{bmatrix}=0.533\,k$$

$$K_2 = \boldsymbol{\phi}_2^T\ \mathbf{k}\ \boldsymbol{\phi}_2 = \begin{bmatrix}-1.429 & 1.000\end{bmatrix}\begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix}-1.429\\1.000\end{bmatrix}=7.942\,k$$

$$\omega_1^2 = K_1 / M_1 = 0.411\,k/m \qquad \omega_2^2 = K_2 / M_2 = 2.700\,k/m$$

Örnek / Etkili modal kütle

$$M_i^* = \frac{(\Sigma_{j=1}^2 \; m_j \; \phi_{ji})^2}{(\Sigma_{j=1}^2 \; m_j \; \phi_{ji}^2)} = \frac{(m_1 \; \phi_{1i} + m_2 \; \phi_{2i})^2}{m_1 \; \phi_{1i}^2 + m_2 \; \phi_{2i}^2}$$

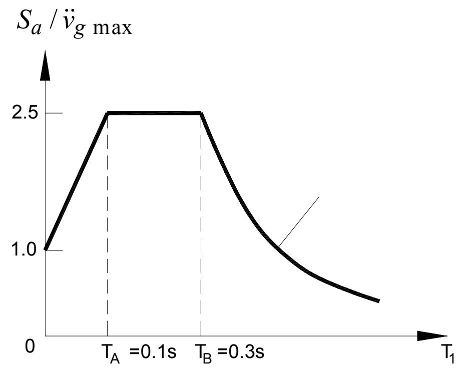
$$M_1^* \; g = \frac{(1.0\times 0.629 + 0.9\times 1.000)^2}{1.0\times 0.629^2 + 0.9\times 1.000^2}\times 147.0 = 265.2kN$$

$$M_2^* \; g = \frac{(-1.0\times 1.429 + 0.9\times 1.000)^2}{1.0\times 1.429^2 + 0.9\times 1.000^2}\times 147.0 = 14.0kN$$

$$M_1^* \; g + M_2^* \; g = 265.2 + 14.0 = 279.1Kn$$

$$m_1 \; g + m_2 \; g = (1.0 + 0.9)\times 147.0 = 279.3kN$$

Örnek
Tasarım spektrumu



Örnek
Modal taban kesme kuvveti

$$\ddot{v}_{g\,\max} = 0.4g$$

$$T_1 = 0.262\,s \qquad S_{a1} = 1.0\,g$$

$$T_2 = 0.102\,s \qquad S_{a2} = 1.0\,g$$

$$V_{b\,1} = M_1^* \; S_{a1} = 265.2kN \qquad V_{b\,2} = M_2^* \; S_{a2} = 14.0kN$$

Örnek / Modal kat kuvvetleri

$$f_{ji} = V_{b\,i}\frac{m_j \; \phi_{ji}}{m_1 \; \phi_{1i} + m_2 \; \phi_{1i}}$$

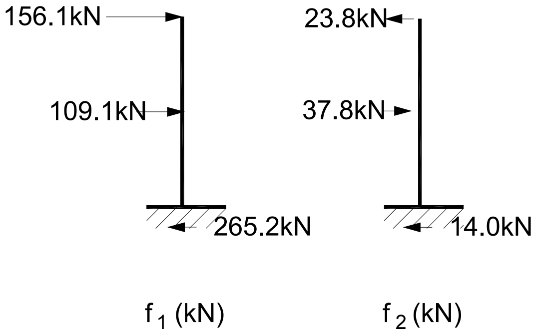
$$f_{11} = 265.2\frac{0.629\times 1.0}{0.629\times 1.0 + 1.000\times 0.9} = 109.1kN$$

$$f_{21} = 265.2\frac{1.000\times 0.9}{0.629\times 1.0 + 1.000\times 0.9} = 156.1kN$$

$$f_{12} = 14.0\frac{-1.429\times 1.0}{-1.429\times 1.0 + 1.000\times 0.9} = 37.8kN$$

$$f_{22} = 14.0\frac{1.000\times 0.9}{-1.429\times 1.0 + 1.000\times 0.9} = -23.8kN$$

Örnek
Modal kat kuvvetleri



Örnek
Kat yerdeğiştirmleri

$$\mathbf{k} = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix} \qquad \mathbf{d} = \begin{bmatrix} 1/k & 1/k \\ 1/k & 2/k \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \mathbf{d}\,\mathbf{f} = \begin{bmatrix} 1/k & 1/k \\ 1/k & 2/k \end{bmatrix} \begin{bmatrix} 109.1 \\ 156.1 \end{bmatrix} = \begin{bmatrix} 12.6mm \\ 20.1mm \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} = \mathbf{d}\,\mathbf{f} = \begin{bmatrix} 1/k & 1/k \\ 1/k & 2/k \end{bmatrix} \begin{bmatrix} 37.8 \\ -23.8 \end{bmatrix} = \begin{bmatrix} 1.7mm \\ -1.2mm \end{bmatrix}$$

Örnek
Birleştirilmiş taban kesme kuvveti ve kat yerdeğiştirmesleri

$$V_b = \sqrt{V_{b1}^2 + V_{b2}^2} = \sqrt{265.2^2 + 14.0^2} = 265.6kN$$

$$v_1 = \sqrt{v_{11}^2 + v_{21}^2} = \sqrt{12.6^2 + 1.7^2} = 12.6mm$$

$$v_2 = \sqrt{v_{21}^2 + v_{22}^2} = \sqrt{20.1^2 + 1.2^2} = 20.1mm$$

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Single-degree-of-freedom system subjected to an earthquake motion:

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g(t) \qquad (7.26)$$

$$\omega_D = \omega\sqrt{(1-\xi^2)} \qquad u(t,\xi,\omega) = -\frac{1}{\omega_D}\int_o^t \ddot{u}_g(\tau)\exp[-\xi\omega(t-\tau)]\sin[\omega_D(t-\tau)]d\tau$$

$$\dot{u}(t,\xi,\omega) = -\int_o^t \ddot{u}_g(\tau)\exp[-\xi\omega(t-\tau)]\cos[\omega_D(t-\tau)]d\tau - \xi\omega u(t,\xi,\omega)$$

$$\ddot{u}(t,\xi,\omega) + \ddot{u}_g(t) = -\omega^2 u(t,\xi,\omega) - 2\xi\omega\dot{u}(t,\xi,\omega) \qquad (7.27)$$

Maximum displacement $S_d(T)$, velocity $S_v(T)$ and acceleration $S_a(T)$

$$S_d(\xi,T) = |u(t,\xi,\omega)|_{\max} \qquad S_v(\xi,T) = |\dot{u}(t,\xi,\omega)|_{\max}$$

$$S_a(\xi,T) = |\ddot{u}(t,\xi,\omega) + \ddot{u}_g(t)|_{\max} \qquad (\omega_D \approx \omega \text{ assumed}) \qquad (7.28)$$

Since the exact initial time of the earthquake is not known and the functions *sin* and *cos* display similar variations, the following relationship can be written:

$$S_v(\xi,T) \approx \left| \int_0^t \ddot{u}_g(\tau) \exp[-\xi\omega(t-\tau)] \sin[\omega(t-\tau)]\, d\tau \right|_{max} \quad (7.29)$$

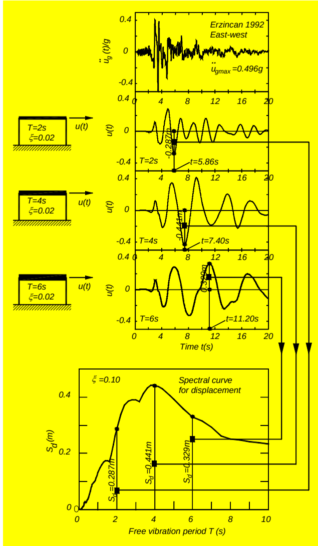
$$S_a(\xi,T) \approx \omega S_v(\xi,T) \approx \omega^2 S_d(\xi,T) \quad (7.30)$$

Elastic and inertia forces:

$$f_{S\,max} = k\,S_d \qquad f_{I\,max} = m\,S_a \quad (7.33)$$

Maximum elastic energy

$$[E(t,\omega)]_{max} = \frac{1}{2} k\, u_{max}^2 = \frac{1}{2} k\, S_d^2 = \frac{1}{2} m\,\omega^2\, S_d^2 = \frac{1}{2} m\, S_v^2 \quad (7.35)$$



Properties of the spectral curves:

$$u(t,T\to 0,\xi)\to 0 \qquad \dot{u}(t,T\to 0,\xi)\to 0 \qquad \ddot{u}(t,T\to 0,\xi)\to 0$$

$$S_d(T\to 0,\xi) = |u(t,T\to 0,\xi)|_{max} \to 0$$

$$S_v(T\to 0,\xi) = |\dot{u}(t,T\to 0,\xi)|_{max} \to 0$$

$$S_v(T\to 0,\xi) = |\ddot{u}(t,T\to 0,\xi) + \ddot{u}_g(t)|_{max} \to |\ddot{u}_g(t)|_{max} \quad (7.36)$$

$$|u(t,T\to\infty,\xi) + u_g(t)| \to 0 \qquad |\dot{u}(t,T\to\infty,\xi) + \dot{u}_g(t)| \to 0$$

$$|\ddot{u}(t,T\to\infty,\xi) + \ddot{u}_g(t)| \to 0$$

$$S_d(T\to\infty,\xi) = |u(t,T\to\infty,\xi)|_{max} \to |-u_g(t)|_{max}$$

$$S_v(T\to\infty,\xi) = |\dot{u}(t,T\to\infty,\xi)|_{max} \to |-\dot{u}_g(t)|_{max}$$

$$S_a(T\to\infty,\xi) = |\ddot{u}(t,T\to\infty,\xi) + \ddot{u}_g(t)|_{max} \to 0 \quad (7.37)$$

$$S_d \approx T\,S_v/(2\pi) \qquad S_a \approx 2\pi\,S_v/T \quad (7.38)$$

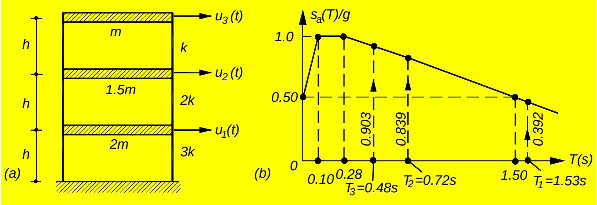
Example

Consider the three-degree-of-freedom system shown and evaluate the equivalent seismic elastic forces $m = 25 \times 10^3\,kg$ $k = 1200\,kN/m$

The mass and stiffness matrices of the system and the free vibration frequencies and the mode shapes:

$$\mathbf{m} = \begin{bmatrix} 2m & 0 & 0 \\ 0 & 1.5m & 0 \\ 0 & 0 & m \end{bmatrix} \quad \mathbf{k} = \begin{bmatrix} 5k & -2k & 0 \\ -2k & 3k & -k \\ 0 & -k & k \end{bmatrix} \quad \mathbf{u}(t) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\left|\mathbf{k} - \omega^2\,\mathbf{m}\right| = 0 \quad \omega_1^2 = 0.351\,k/m \quad \omega_2^2 = 1.607\,k/m \quad \omega_3^2 = 3.542\,k/m$$



$$\boldsymbol{\phi}_1 = \begin{bmatrix} 0.302 \\ 0.648 \\ 1.000 \end{bmatrix} \quad \boldsymbol{\phi}_2 = \begin{bmatrix} -0.679 \\ -0.606 \\ 1.000 \end{bmatrix} \quad \boldsymbol{\phi}_3 = \begin{bmatrix} 2.438 \\ -2.541 \\ 1.000 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} h \\ 2h \\ 3h \end{bmatrix}$$

The generalized masses and stiffnesses:

$$M_1 = \boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_1 = 1.812\,m \quad M_2 = 2.743\,m \quad M_3 = 22.573\,m$$

$$K_1 = \boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_1 = 0.637\,k \quad K_2 = 3.973\,k \quad K_3 = 79.951\,k$$

$$Y_1(\tau) = \boldsymbol{\phi}_1^T\,\mathbf{m}\,\mathbf{u}/M_1 = 0.333\,u_1(t) + 0.536\,u_2(t) + 0.552\,u_3(t)$$

$$Y_2(t) = -0.549\,u_1(t) - 0.368\,u_2(t) + 0.404\,u_3(t)$$

$$Y_3(t) = 0.216\,u_1(t) - 0.169\,u_2(t) + 0.044\,u_3(t)$$

$$L_1 = \boldsymbol{\phi}_1^T\,\mathbf{m}\,\mathbf{1} = \begin{bmatrix} 0.302 \\ 0.648 \\ 1.000 \end{bmatrix}^T \begin{bmatrix} 2m & 0 & 0 \\ 0 & 1.5m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2.576\,m$$

$$L_2 = \boldsymbol{\phi}_2^T\,\mathbf{m}\,\mathbf{1} = 1.267\,m \quad L_3 = \boldsymbol{\phi}_3^T\,\mathbf{m}\,\mathbf{1} = 2.065\,m$$

$$\Gamma_1 = \frac{L_1}{M_1} = \frac{1.267\,m}{1.812\,m} = 1.422 \quad \Gamma_2 = -0.512 \quad \Gamma_3 = 0.091$$

$$M_1^* = L_1\,\Gamma_1 = 2.576 \times 1.422\,m = 3.663\,m \quad M_2^* = 0.649\,m \quad M_3^* = 0.188\,m$$

$$L_1^\theta = \sum_{n=1}^{3} m_n\,\phi_{n1}\,h_n = m\,h\,(2.0 \times 0.302 \times 1 + 1.5 \times 0.648 \times 2 + 1.0 \times 1.000 \times 3) = 5.548\,m\,h$$

$$L_2^\theta = -0.176\,m\,h \quad L_3^\theta = 0.253\,m\,h$$

$$h_1^* = \frac{L_1^\theta}{L_1} = \frac{5.548\,m\,h}{2.576\,m} = 2.154\,h \quad h_2^* = 0.139\,h \quad h_3^* = 0.123\,h$$

The sum of the modal masses is equal to the sum of the actual masses:

$$\sum_{j=1}^{3} m_j = (2.0 + 1.5 + 1.0)\,m = 4.5m = \sum_{j=1}^{3} M_j^* = (3.663 + 0.649 + 0.188)\,m$$

The sum of the moments of the story masses with respect to the base is equal to the moment of the effective modal masses by using the effective heights.

$$\sum_{j=1}^{3} m_j\,h_j = (2.0 \times 1 + 1.5 \times 2 + 1.0 \times 3)\,m\,h = 8\,m\,h$$

$$\sum_{j=1}^{3} h_j^*\,M_j^* = (3.663 \times 2.154 + 0.649 \times 0.139 + 0.188 \times 0.123) = 8\,m\,h$$

The periods are calculated for using the curve of the spectral acceleration:

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{0.351k/m}} = \frac{2\pi}{\sqrt{0.351\times 1.2\times 10^6/(25\times 10^3)}} = 1.53s$$

$$T_2 = 0.72s \qquad T_3 = 0.48s$$

The spectral accelerations for each periods:

$$S_a(T_1 = 1.53s) = 0.392\,g \qquad S_a(T_2 = 0.72s) = 0.839\,g$$

$$S_a(T_3 = 0.48s) = 0.903\,g$$

Modal base shear forces for each mode:

$$|V_{b1}|_{\max} = M_1^* \, S_a(T_1) = 3.663\times 0.392\,m\,g = 1.436\,m\,g$$

$$|V_{b2}|_{\max} = M_2^* \, S_a(T_2) = 0.649\times 0.839\,m\,g = 0.545\,m\,g$$

$$|V_{b3}|_{\max} = M_3^* \, S_a(T_3) = 0.188\times 0.903\,m\,g = 0.170\,m\,g$$

Corresponding story forces for the each mode:

$$|f_{nj}|_{\max} = |V_{bj}|_{\max} \frac{m_n\,\phi_{nj}}{\sum_{k=1}^{3} m_k\,\phi_{kj}}$$

$$|f_{11}|_{\max} = 1.436\,m\,g \frac{2\times 0.302}{2\times 0.302 + 1.5\times 0.648 + 1\times 1.000} = 0.337\,m\,g$$

$$|f_{21}|_{\max} = 1.436\,m\,g \frac{1.5\times 0.648}{2\times 0.302 + 1.5\times 0.648 + 1\times 1.000} = 0.542\,m\,g$$

$$|f_{31}|_{\max} = 1.436\,m\,g \frac{1\times 1.000}{2\times 0.302 + 1.5\times 0.648 + 1\times 1.000} = 0.557\,m\,g$$

$$|f_{12}|_{\max} = 0.545\,m\,g \frac{-2\times 0.679}{-2\times 0.679 - 1.5\times 0.606 + 1\times 1.000} = 0.583\,m\,g$$

$$|f_{22}|_{\max} = 0.390\,m\,g \qquad |f_{32}|_{\max} = -0.430\,m\,g$$

$$|f_{13}|_{\max} = 0.170\,m\,g \frac{2\times 2.438}{2\times 2.438 - 1.5\times 2.541 + 1\times 1.000} = 0.401\,m\,g$$

$$|f_{23}|_{\max} = -0.313\,m\,g \qquad |f_{33}|_{\max} = 0.082\,m\,g$$

Corresponding modal displacements

$$\mathbf{d} = \mathbf{k}^{-1} = \frac{1}{6k}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}$$

$$\mathbf{u}_{1\max} = \mathbf{d}\mathbf{f}_{1\max} = \frac{1}{6k}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 5 \\ 2 & 5 & 11 \end{bmatrix}\begin{bmatrix} 0.337mg \\ 0.542mg \\ 0.557mg \end{bmatrix} = \begin{bmatrix} 0.479mg/k \\ 1.028mg/k \\ 1.585mg/k \end{bmatrix}$$

$$\mathbf{u}_{2\max} = \mathbf{d}\mathbf{f}_{2\max} = \mathbf{d}\begin{bmatrix} 0.583mg \\ 0.390mg \\ -0.430mg \end{bmatrix} = \begin{bmatrix} -0.269mg/k \\ 0.161mg/k \\ 0.181mg/k \end{bmatrix}$$

$$\mathbf{u}_{3\max} = \mathbf{d}\mathbf{f}_{3\max} = \mathbf{d}\begin{bmatrix} 0.401mg \\ -0.313mg \\ -0.082mg \end{bmatrix} = \begin{bmatrix} 0.023mg/k \\ -0.059mg/k \\ 0.057mg/k \end{bmatrix}$$

Modal combination of the story shear by using the rule of the square root of the sum of the squares of the parameters:

$$V_1 = \sqrt{V_{11}^2 + V_{21}^2 + V_{31}^2} = m\,g\sqrt{(1.436)^2 + (0.544)^2 + (0.168)^2} = 1.545\,m\,g$$

$$V_2 = \sqrt{V_{12}^2 + V_{22}^2 + V_{32}^2} = m\,g\sqrt{(1.099)^2 + (0.040)^2 + (0.401)^2} = 1.171\,m\,g$$

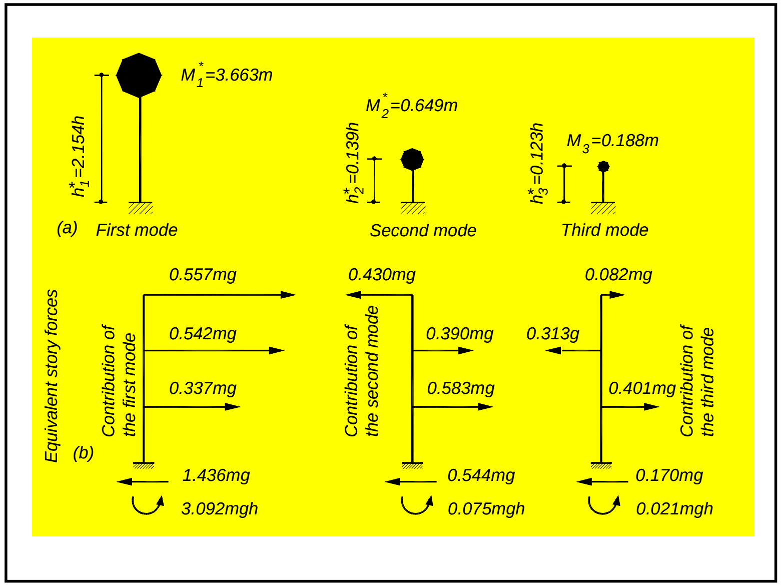
$$V_3 = \sqrt{V_{13}^2 + V_{23}^2 + V_{33}^2} = m\,g\sqrt{(0.577)^2 + (0.430)^2 + (0.082)^2} = 0.724\,m\,g$$

The modal combination of the modal displacements:

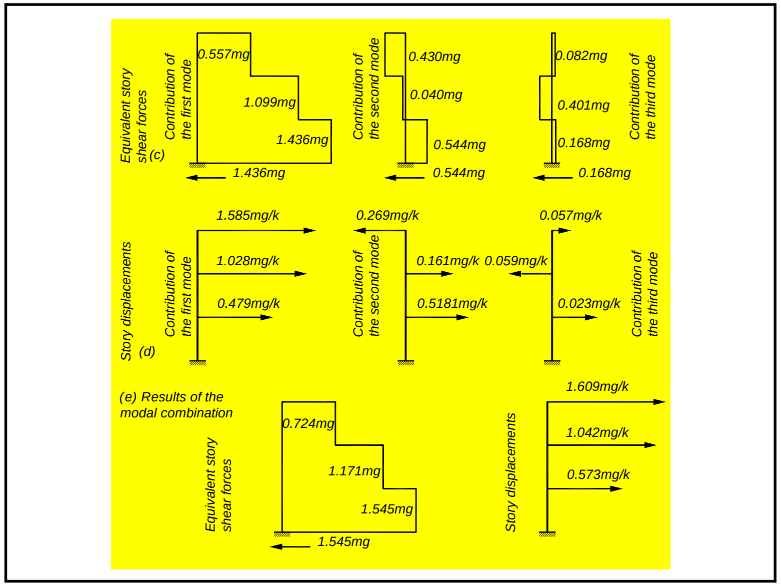
$$u_1 = \sqrt{u_{11}^2 + u_{21}^2 + u_{31}^2} = (m\,g/k)\sqrt{(0.479)^2 + (0.181)^2 + (0.023)^2} = 0.513\,m\,g/k$$

$$u_2 = \sqrt{u_{12}^2 + u_{22}^2 + u_{32}^2} = (m\,g/k)\sqrt{(1.028)^2 + (0.161)^2 + (0.059)^2} = 1.042\,m\,g/k$$

$$u_3 = \sqrt{u_{13}^2 + u_{23}^2 + u_{33}^2} = (m\,g/k)\sqrt{(1.585)^2 + (0.269)^2 + (0.057)^2} = 1.609\,m\,g/k$$



M1*=3.663m
h1*=2.154h
M2*=0.649m
h2*=0.139h
M3=0.188m
h3*=0.123h
(a)
First mode
Second mode
Third mode
Equivalent story forces
(b)
Contribution of the first mode
0.557mg
0.542mg
0.337mg
1.436mg
3.092mgh
Contribution of the second mode
0.430mg
0.390mg
0.583mg
0.544mg
0.075mgh
Contribution of the third mode
0.082mg
0.313g
0.401mg
0.170mg
0.021mgh



Equivalent story shear forces (c)
Contribution of the first mode
0.557mg
1.099mg
1.436mg
1.436mg
Contribution of the second mode
0.430mg
0.040mg
0.544mg
0.544mg
Contribution of the third mode
0.082mg
0.401mg
0.168mg
0.168mg
Story displacements (d)
Contribution of the first mode
1.585mg/k
1.028mg/k
0.479mg/k
Contribution of the second mode
0.269mg/k
0.161mg/k
0.5181mg/k
Contribution of the third mode
0.057mg/k
0.059mg/k
0.023mg/k
(e) Results of the modal combination
Equivalent story shear forces
0.724mg
1.171mg
1.545mg
1.545mg
Story displacements
1.609mg/k
1.042mg/k
0.573mg/k