ADVANCED DYNAMICS OF STRUCTURES / Home Work 3 / November 19, 2014

Problem # 1:

Consider the column which can be represented as a system of three degrees-of-freedom shown:

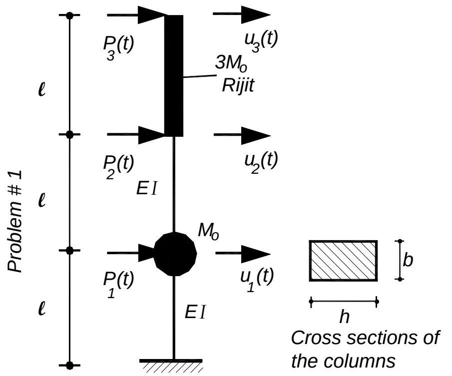
a. Write down the equations of motion of the system by including the external loads. Evaluate the mass matrix $\mathbf{m}$, the rigidity matrix $\mathbf{k}$ **,** and the flexibility matrix $\mathbf{k}=\mathbf{d}^{-1}$,

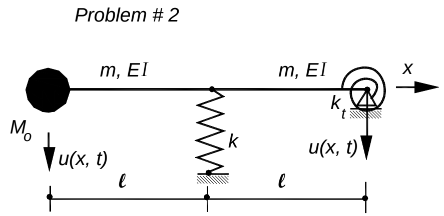
b. Determine the three circular frequencies and the periods of the free vibration ω_i and T_i and the corresponding mode shapes ϕ_i . Give their graphical representation ($i=1,2,3$),

a. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\phi_1^T\mathbf{m}\phi_2$, $\phi_1^T\mathbf{m}\phi_3$, $\phi_2^T\mathbf{m}\phi_3$ and $\phi_1^T\mathbf{k}\phi_2$, $\phi_1^T\mathbf{k}\phi_3$, $\phi_1^T\mathbf{k}\phi_3$,

b. Evaluate the generalized masses and stiffness $M_i=\phi_i^T\mathbf{m}\phi_i$, and $K_i=\phi_i^T\mathbf{k}\phi_i$, and assess $\omega_i^2=K_i/M_i$ ($i=1,2,3$),

c. The heights of the stories are $\ell=3.0meter$, the columns have a cross section of $0.40m\times0.80m$, the first period of the system is $T_1=0.30s$ and $E=30GPa$. Find the numerical values of the mass M_o , the second period T_2 and the third period T_3 of the system.





Problem # 2:

Consider a beam of length 2ℓ supported at the two points as a distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section of the beam. The beam has a lumped mass of M_o at the right end of the beam.

a. Write down the boundary conditions for the free vibration of the beam.

b. By assuming $M_o=m\ell$, $k_v=EI/\ell^3$ and $k_t=EI/\ell$ obtain the frequency determinant and first two circular frequencies $\beta_i^4=\dfrac{m\,\ell^4\,\omega_i^2}{EI}$ for $i=1,2$.

ADVANCED DYNAMICS OF STRUCTURES / Home Work 4 / December 10, 2014

Consider the system of three degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the ground motion $u_g(t)$ and evaluate the mass matrix $\mathbf{m}$, the rigidity matrix $\mathbf{k}$ **,** and the flexibility matrix $\mathbf{k}=\mathbf{d}^{-1}$,

b. Determine the three circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI , M and ℓ . Obtain the corresponding mode shapes ϕ_i and give their graphical representations ($i=1,2,3$),

c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\phi_1^T\mathbf{m}\phi_2$, $\phi_1^T\mathbf{m}\phi_3$, $\phi_2^T\mathbf{m}\phi_3$ and $\phi_1^T\mathbf{k}\phi_2$, $\phi_1^T\mathbf{k}\phi_3$, $\phi_1^T\mathbf{k}\phi_3$,

d. Evaluate the generalized masses and stiffness $M_i=\phi_i^T\mathbf{m}\phi_i$, and $K_i=\phi_i^T\mathbf{k}\phi_i$, and assess $\omega_i^2=K_i/M_i$ ($i=1,2,3$),

e. The heights of the stories are $\ell=3meter$, the columns have cross section of $b/h=0.40m/0.70m$, the first period of the system is $T_1=0.30s$ and $E=30GPa$. Find the numerical values the parameter M, the second period T_2 and the third period T_3 of the system.

f. Determine the effective modal masses M_1^* , M_2^* and M_3^* and assess that $M_1^*+M_2^*+M_3^*=5M$

g. Evaluate the base shear forces V_{b1} , V_{b2} and V_{b3} corresponding to the three mode shapes, the equivalent forces applied to the system at the story levels for both cases, the story shear forces and the story displacements by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.