ADVANCED DYNAMICS OF STRUCTURES / Homework 1 / October 21, 2008

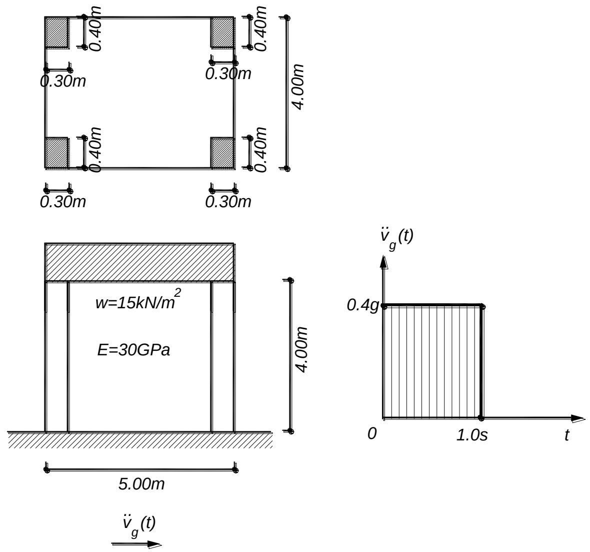
Problem # 1

a. A single-degree-of-freedom system having a mass m, a lateral stiffness k and a damping c is subjected to a ground motion defined as

$$\ddot{v}_g(t) = 0.4g \quad \text{for } 0 \le t \le t_1, \qquad \ddot{v}_g(t) = 0 \quad \text{for } t_1 \le t,$$

obtain the lateral displacement $v(t)$ for $0 \le t \le t_1$ and for $t_1 \le t$ separately. Determine the integration coefficients by assuming $v(t=0)=0$ and $\dot{v}(t=0)=0$.

b. Consider the single-degree-of-freedom system shown, evaluate the mass M, the lateral stiffness K, the period T, the frequency f and the circular frequency ω. Obtain its lateral displacement $v(t)$ for $0 \le t \le 1s$ and $v(t)$ for $t_1 \le 2s$ separately by assuming that it is subjected to a ground motion as shown and by assuming the motion starts from the rest position of the system and draw its variations for $\xi = 0.0$, 0.10 and 0.20.



Problem # 2

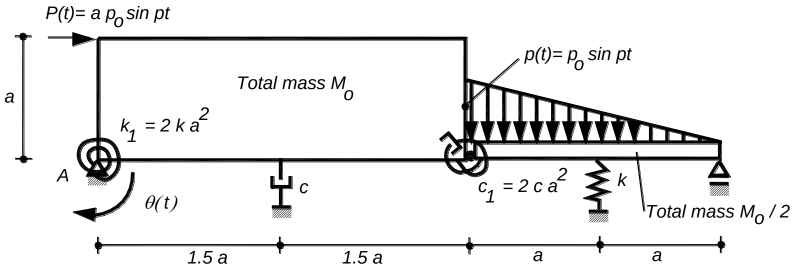
For the rigid-body assemblage shown,

a. Set up the equation of motion for the rotation angle $\theta(t)$ of the point A by using the principle of the virtual work.

b. By assuming $k_1 = 2\,k\,a^2$ determine the period of the system as $T = \alpha\sqrt{M_o / k}$ and evaluate α.

c. By assuming $c_1 = 2\,c\,a^2$ determine the effective damping coefficient of the system as $c_{effective} = \beta\,c$ and evaluate β and $\xi = c_{effective} / (2\,m\,\omega)$.

d. For $\xi = 0.10$, $(p_o\,T^2)/M_o = 2.0$ and $p = 0.9\omega = 0.9 \times 2\pi / T$ draw the time variation of $\theta(t/T)$ for $0 \le t/T \le 4$ under the assumption of the homogeneous initial conditions $\theta(t/T=0)=0$ $\quad \dot{\theta}(t/T=0)=0$.



Problem # 3

a. Obtain the undamped free vibration period T of the single-degree-of-freedom system shown in the figure. By assuming $W_o = M_o\,g = 150kN$, and $K_o = 1000kN/m$

b. Evaluate the displacement history of the system subjected to an external load $P(t)$ by using step by step numerical integration of the Duhamel integral for $0 \le t \le t_1$ by using Simpson rule under the assumption of the homogeneous initial conditions $v(t=0)=0$ $\quad \dot{v}(t=0)=0$.

$t_o = 0.06s \quad t_1 = 1.0s \quad \Delta\tau = 0.01s$

$$v(t) = \frac{1}{M_o\,\omega}\int_o^t P(\tau)\sin\omega(t-\tau)\,d\tau \qquad 0 \le t \le t_o$$

$$v(t) = \frac{1}{M_o\,\omega}\int_o^{t_o} P(\tau)\sin\omega(t-\tau)\,d\tau \qquad t_o \le t \le t_1$$

t(s)	P(kN)
0	0
0.01	35.6
0.02	71.1
0.03	80.0
0.04	71.1
0.05	35.6
0.06	0
0.08	0
0.10	0
1.00	0