## ADVANCED DYNAMICS OF S TRUCTURES / Home Work 2 / November 25, 2009

 H.Boduroğlu / Z. CelepConsider the column which can be represented as a system of three degrees-of-freedom shown:
a. Write down the equations of motion of the system shown by including the ground motion $v_{g}(t)$ and the external load $p_{i}(t)$ and evaluate the mass matrix $\mathbf{m}$, the rigidity matrix $\mathbf{k}$, and the fle xibility matrix $\mathbf{k}=\mathbf{d}^{-1}$,
b. Determine the three circu lar frequencies and the periods of the free vibration $\omega_{i}$ and $T_{i}$ and the corresponding mode shapes $\phi_{i}$. Give their graphical representation $(i=1,2,3)$,
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\phi_{1}{ }^{\mathrm{T}} \mathrm{m} \phi_{2}, \phi_{1}{ }^{\mathrm{T}} \mathrm{m} \phi_{3}$ and $\phi_{1}{ }^{\mathrm{T}} \mathbf{k} \phi_{2}, \phi_{1}{ }^{\mathrm{T}} \mathbf{k} \phi_{3}$,
d. Evaluate the generalized masses and stiffness $M_{i}=\phi_{\mathbf{i}}{ }^{\mathbf{T}} \mathbf{m} \phi_{\mathrm{i}}$, and $K_{i}=\phi_{\mathbf{i}}{ }^{\mathbf{T}} \mathbf{k} \phi_{\mathbf{i}}$, and assess $\omega_{i}{ }^{2}=K_{i} / M_{i}$.
e. Obtain the first free vibration mode shapes $\phi_{1}$ and the corresponding the first circular frequencies $\omega_{i}$ of the system by using Stodola method or the inverse iteration method with shift.
f. The heights of the stories are $h=3$ meter, the colu mn has a cross section of $0.50 \mathrm{~m} \times 0.90 \mathrm{~m}$, the first period of the system is $T_{l}=0.25 \mathrm{~s}$ and $E=30 \mathrm{GPa}$. Find the numerical values of the mass $m$, the second period $T_{2}$ and the third period $T_{3}$ of the system.

$h_{c}$

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b. Determine the three circular frequencies and the periods of the free vibration $\omega_{i}$ and $T_{i}$ and the corresponding mode shapes $\phi_{i}$. Give their graphical representation $(i=1,2,3)$,
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e. Obtain the first free vibration mode shapes $\phi_{1}$ and the corresponding the first circular frequencies $\omega_{i}$ of the system by using Stodola method,
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Column section


