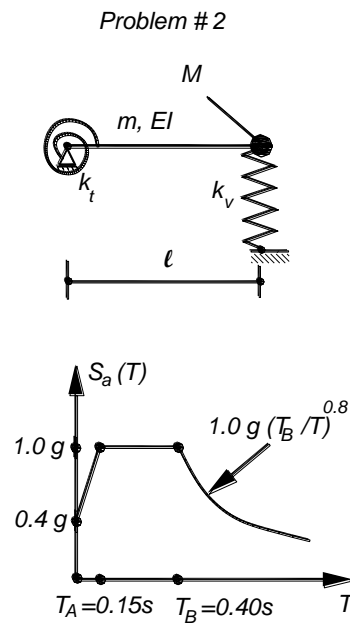
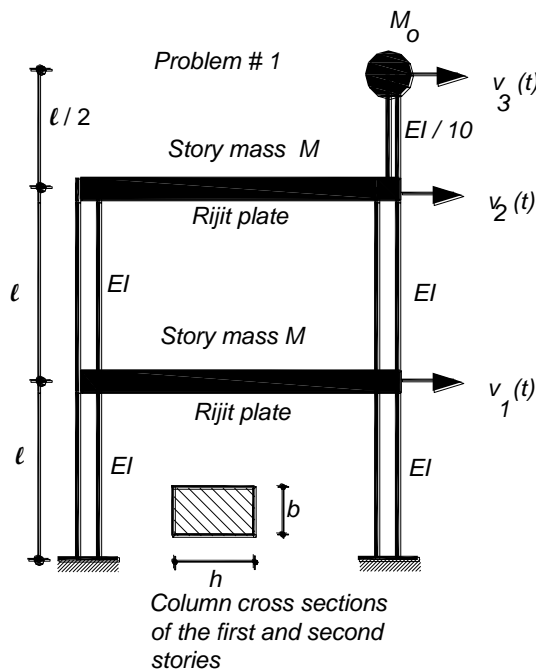


**Problem #1** (to be collected at the Final Exam):

Consider the system of three degrees-of-freedom shown:

- Write down the equations of motion of the system by including the ground motion  $v_g(t)$  and evaluate the mass matrix  $\mathbf{m}$ , the rigidity matrix  $\mathbf{k}$ , and the flexibility matrix  $\mathbf{k} = \mathbf{d}^{-1}$ ,
- Assuming  $M_o = M / 4$ , determine the three circular frequencies and the periods of the free vibration  $\omega_i$  and  $T_i$  in terms of  $EI$ ,  $M$  and  $\ell$ . Obtain the corresponding mode shapes  $\phi_i$  and give their graphical representation ( $i = 1, 2, 3$ ),
- Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix  $\phi_1^T \mathbf{m} \phi_2$ ,  $\phi_1^T \mathbf{m} \phi_3$  and  $\phi_1^T \mathbf{k} \phi_2$ ,  $\phi_1^T \mathbf{k} \phi_3$ ,
- Evaluate the generalized masses and stiffness  $M_i = \phi_i^T \mathbf{m} \phi_i$ , and  $K_i = \phi_i^T \mathbf{k} \phi_i$ , and assess  $\omega_i^2 = K_i / M_i$ . ( $i = 1, 2, 3$ ),
- Obtain the first free vibration mode shapes  $\phi_1$  and the corresponding the first circular frequencies  $\omega_1$  of the system by using Stodola method,
- The heights of the stories are  $\ell = 3\text{meter}$ , the columns have cross section of  $0.30\text{m} \times 0.50\text{m}$ , the first period of the system is  $T_1 = 0.25\text{s}$  and  $E = 30\text{GPa}$ . Find the numerical values the parameter  $M$  and  $M_o$ , the second  $T_2$  and the third  $T_3$  period of the system.
- Determine the effective modal masses  $M_1^*$ ,  $M_2^*$  and  $M_3^*$  and asses that  $M_1^* + M_2^* + M_3^* = 2M + M_o$
- Evaluate the base shear forces  $V_{b1}$ ,  $V_{b2}$  and  $V_{b3}$  corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases, the story shear forces and the story displacements by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.



**Problem #2** (to be collected December 17, 2008):

Consider the distributed parameter system shown where  $m$  is the mass per unit length and  $EI$  is the bending rigidity of the cross section. The beam has a lumped mass of  $M$  at the right end.

- Write down the boundary conditions for the free vibration of the beam.
- By assuming  $M = m \ell$ ,  $k_v = EI / \ell^3$  and  $k_t = EI / \ell$  obtain the frequency determinant and first two

circular frequencies  $\alpha_i^4 = \frac{m \ell^4 \omega_i^2}{EI}$  for  $i = 1, 2$ .