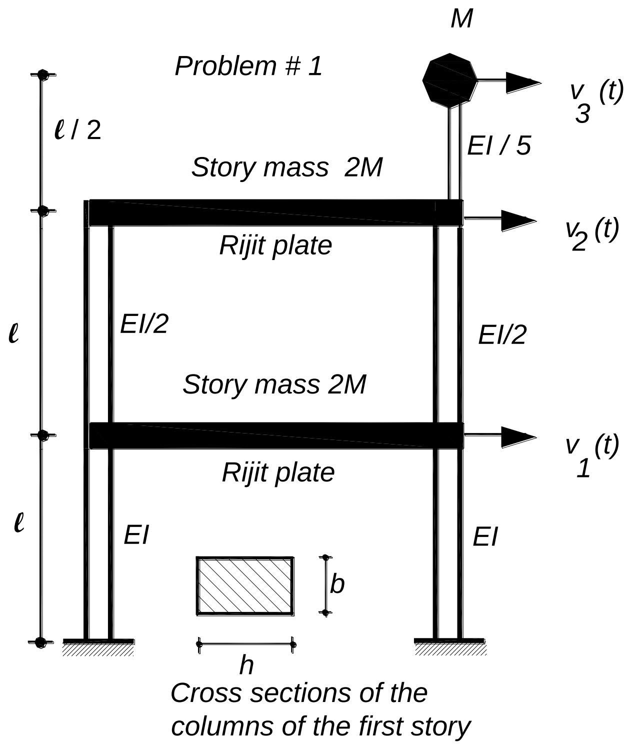
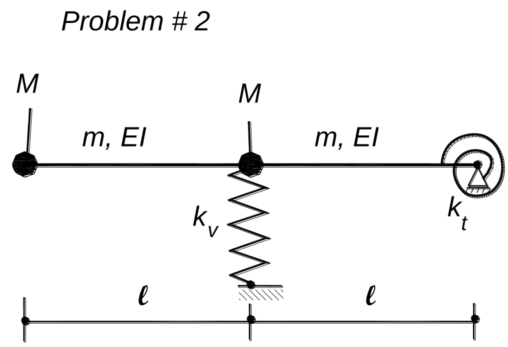
ADVANCED DYNAMICS OF STRUCTURES / Home Work 3 / December 16, 2009
(to be collected at the Final Exam) *HBoduroğlu/ZCelep*

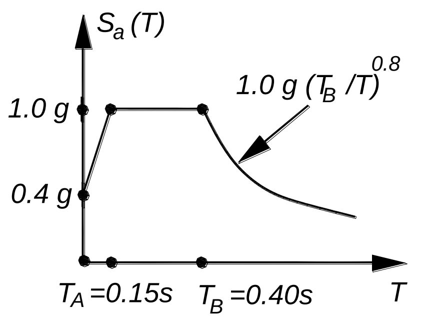
***Problem # 1*:**

Consider the system of three degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the ground motion $v_g(t)$ and evaluate the mass matrix **m**, the rigidity matrix **k,** and the flexibility matrix $\mathbf{k} = \mathbf{d}^{-1}$,
b. Determine the three circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, M and ℓ. Obtain the corresponding mode shapes ϕ_i and give their graphical representations ($i = 1, 2, 3$),
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\phi_1^T \mathbf{m}\, \phi_2$, $\phi_1^T \mathbf{m}\, \phi_3$, $\phi_2^T \mathbf{m}\, \phi_3$ and $\phi_1^T \mathbf{k}\, \phi_2$, $\phi_1^T \mathbf{k}\, \phi_3$, $\phi_2^T \mathbf{k}\, \phi_3$,
d. Evaluate the generalized masses and stiffness $M_i = \phi_i^T \mathbf{m}\, \phi_i$, and $K_i = \phi_i^T \mathbf{k}\, \phi_i$, and assess $\omega_i^{\,2} = K_i / M_i$. ($i = 1, 2, 3$),
e. Obtain the first free vibration mode shapes ϕ_1 and the corresponding the first circular frequencies ω_1 of the system by using Stodola method,
f. The heights of the stories are $\ell = 3 meter$, the columns have cross section of $b / h = 0.30m \times 0.60m$, the first period of the system is $T_1 = 0.25s$ and $E = 30GPa$. Find the numerical values the parameter M, the second period T_2 and the third period T_3 of the system.
g. Determine the effective modal masses M_1^*, M_2^* and M_3^* and assess that $M_1^* + M_2^* + M_3^* = 5M$
h. Evaluate the base shear forces V_{b1}, V_{b2} and V_{b3} corresponding to the three mode shapes, the equivalent forces applied to the system at the story levels for both cases, the story shear forces and the story displacements by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.







Problem # 2:

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has two lumped masses of M at the right end and at its middle.

a. Write down the boundary conditions for the free vibration of the beam.

By assuming $M = m\,\ell$, $k_v = E\,I / \ell^3$ and $k_t = E\,I / \ell$ obtain the frequency determinant and first two circular frequencies

$$\beta_i^4 = \frac{m\,\ell^4\,\omega_i^2}{EI} \text{ for } i = 1,2 .$$