ADVANCED DYNAMICS OF STRUCTURES / HOMEWORK / October 3, 2004

- a. Obtain the undamped and damped free vibration period of the single-degree-of-freedom system shown in the figure. W = mg = 200kN k = 1000kN/m $\xi = 0.05$
- b. Evaluate the displacement history of the system subjected to the ground motion history $\ddot{v}_g(t)$ by using step by step numerical integration of the Duhamel integral for $0 \le t \le t_1$ by using Simpson rule under the assumption of the homogeneous initial conditions v(t=0)=0 $\dot{v}(t=0)=0$.

$$t_o = 0.08s$$
 $t_1 = 2.0s$ $\Delta \tau = 0.01s$

$$v(t) = -\frac{1}{\omega_D} \int_0^t \exp\left[-\xi \omega(t-\tau)\right] \ddot{v}_g(\tau) \sin \omega_D(t-\tau) d\tau \qquad 0 \le t \le t_o$$

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t(s)	$\ddot{v}_g(t)/g$
0	0
0.01	0.356
0.02	0.711
0.03	0.800
0.04	0.711
0.05	0.356
0.08	0
0.10	0
0.20	0
0.40	0
2.00	0

c. Obtain the response of the system under the assumption that the ground motion $\ddot{v}_g(t)$ can be represented as a impulse loading. Draw the variation of v(t) for the two last cases.

$$v(t) = \frac{I}{m \omega_D} \exp[-\xi \omega t] \sin \omega_D t \qquad I = -\int_0^t m \ddot{v}_g(\tau) d\tau$$

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