ADVANCED DYNAMICS OF STRUCTURES   October, 2013

Problem # 1

Consider the system of two degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the external loads.

b. Determine the two circular frequencies and the periods of the free vibration ω_i and T_i and the corresponding mode shapes ϕ_i. Give their graphical representation ($i = 1, 2$),

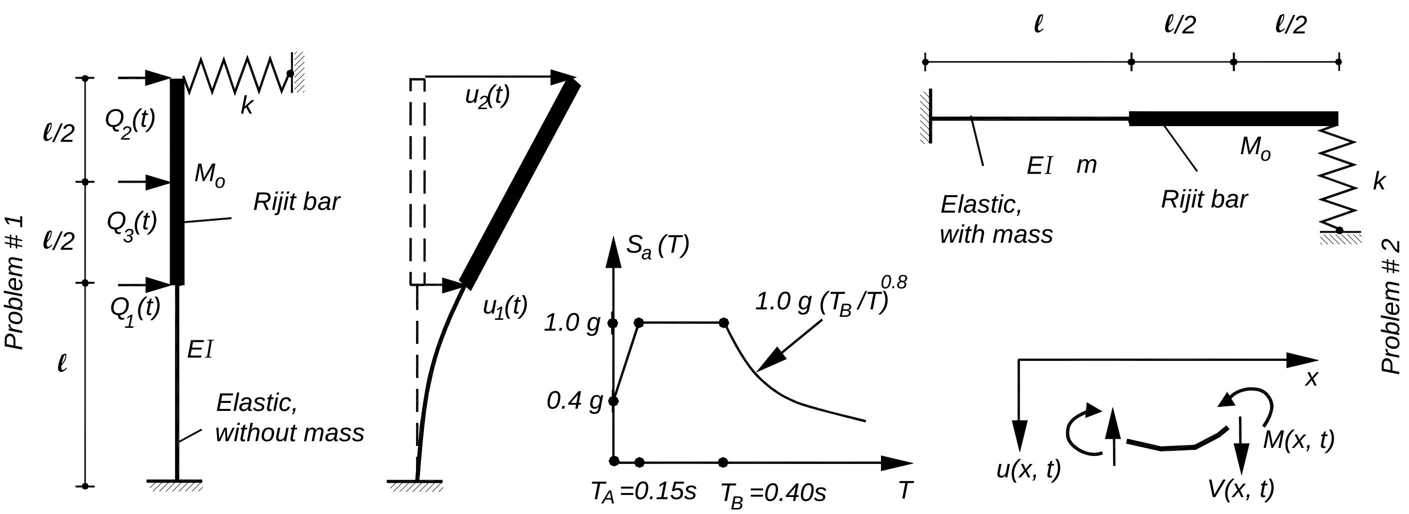
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$,

d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess $\omega_i^{\,2} = K_i / M_i$ for $i = 1, 2$,

e. The height of the column $\ell = 3 meter$ and their cross section of $b / h = 0.25m \times 0.50m$, the first period of the system is $T_1 = 0.20s$ and $E = 30GPa$. Find the numerical values the parameter M_o and the second period T_2 of the system.

f. Determine the effective modal masses M_1^* and M_2^* and assess that $M_1^* + M_2^* = M_o$

g. Evaluate the base shear forces V_{b1} and V_{b2} and the overturning moments M_{b1} and M_{b2} corresponding to the two mode shapes. Obtain the base shear force V_b and the overturning moment M_b by using the SRSS combination rule.



Problem # 2

Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant by assuming $M_O = m\, a$, $k = E\, I / \ell^3$ in terms of β where $\beta^4 = \dfrac{m\, \ell^4\, \omega^2}{EI}$

$\mathbf{m}\, \ddot{\mathbf{u}}(t) + \mathbf{k}\, \mathbf{u}(t) = \mathbf{p}(t) \qquad \mathbf{u}(t) = \left[u_1(t) \quad u_2(t)\right]^T \qquad \mathbf{p}(t)^T = \left[P_1(t) \quad P_2(t)\right] \qquad (\mathbf{k} - \omega_i^2\, \mathbf{m})\, \phi_i = 0 \qquad \omega_i = 2\,\pi / T_i$

$(\mathbf{I} - \omega_i^2\, \mathbf{d}\, \mathbf{m})\, \phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\, \mathbf{m}\right| = 0 \qquad K_i = \phi_i^T \mathbf{k}\, \phi_i \qquad M_i\, \ddot{Y}_i(t) + K_i\, Y_i(t) = \phi_i^T \mathbf{p}(t) \qquad \mathbf{u}(x,t) = \sum \phi_i(x)\, Y_i(t)$

$\ddot{Y}_i(t) + \omega_i^2\, Y_i(t) = 0 \qquad Y_i(t) = \dfrac{\sin \omega_i t}{M_i\, \omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\, d\tau\right] \qquad \dot{Y}_i(t) = \phi_i^T\, \mathbf{m}\, \mathbf{v} / M_i$

$M_i = \phi_i^T \mathbf{m}\, \phi_i \qquad M(x,t) = -EI\, \dfrac{\partial^2 u}{\partial x^2} \qquad V(x,t) = -EI\, \dfrac{\partial^3 u}{\partial x^3} \qquad a^4 = \dfrac{m\, \omega^2}{EI}$

$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad L_i = \phi_i^T \mathbf{m}\, \mathbf{1} \qquad \Gamma_i = L_i \,/\, M_i \qquad M_i^* = \Gamma_i\, L_i$

$I_\theta = \dfrac{M}{12}(a^2 + b^2) \qquad V_{b\,j} = M_j^*\, A_{j\,max} \qquad f_{ij} = V_{b\,j}\, \dfrac{m_i\, \phi_{ij}}{\sum_{l=1}^{n} m_l\, \phi_{lj}} \qquad M_j^* = \dfrac{\left(\sum_{i=1}^{n} m_i\, \phi_{ij}\right)^2}{\sum_{i=1}^{n} m_i\, \phi_{ij}^2}$

ZCelep