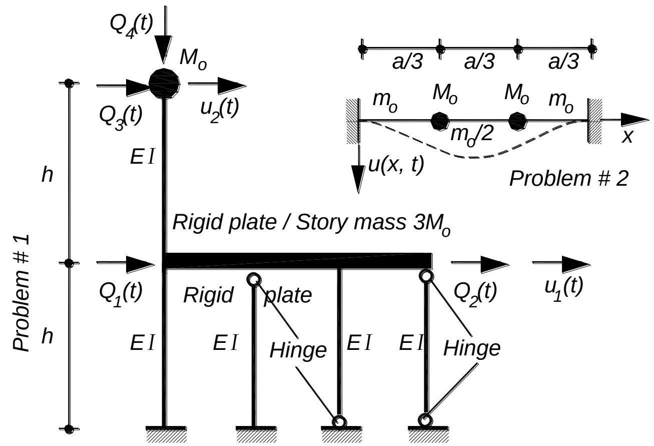
ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / November 26, 2014

Problem # 1:

Consider the system of two degrees-of-freedom shown where the first story is rigid plate having a mass of $3M_o$ and the second story consists of a cantilever column with a lumped mass of M_o at its tip. (a) Evaluate the mass matrix $\mathbf{m}$, the flexibility matrix $\mathbf{d}$, the stiffness matrix $\mathbf{k}$ and the load vector $\mathbf{p}$ of the system. (b). Determine circular frequencies and periods of the free vibration ω_i and T_i in terms of EI, m and h. (c). Obtain corresponding two mode shapes ϕ_i and give their graphical representation ($i=1,2$). (d). Check the orthogonality of the mode shapes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_2$. (e) Evaluate the generalized masses and stiffness $M_i=\boldsymbol{\phi}_i^T\,\mathbf{m}\,\boldsymbol{\phi}_i$ and $K_i=\boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2=K_i/M_i$ ($i=1,2$).

Problem # 2:

Consider the elastic beam shown having a mass per unit length $m_o/2$ at its one-third length and m_o at its two-third length. The beam has two lumped masses M_o additionally. (a) Select one of the displacement functions $\psi_i(x)$ by giving why this function is selected. (b) Obtain first circular frequency of the beam by using Rayleigh ratio.



$\psi_1(x)=x^2$,

$\psi_2(x)=x(a-x)$, $\psi_3(x)=x^2(a-x)^2$, $\psi_4(x)=\sin(\pi x/a)$,

$\psi_5(x)=\sin^2(\pi x/a)$, $\psi_6(x)=\sin(2\pi x/a)$, $\psi_7(x)=\sin^2(2\pi x/a)$

$$\mathbf{m}\,\ddot{\mathbf{u}}(t)+\mathbf{k}\,\mathbf{u}(t)=\mathbf{p}(t)\quad \mathbf{u}(t)=\begin{bmatrix}u_1(t) & u_2(t)\end{bmatrix}^T\quad \mathbf{p}(t)^T=\begin{bmatrix}P_1(t) & P_2(t)\end{bmatrix}\quad \omega_i=2\,\pi/T_i$$

$$(\mathbf{k}-\omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i=\mathbf{0}\quad (\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i=\mathbf{0}\quad \left|\mathbf{k}-\omega_i^2\,\mathbf{m}\right|=0\quad \left|\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m}\right|=0\quad M_i=\boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i=\boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i\quad M_i\,\ddot{Y}_i(t)+K_i\,Y_i(t)=\boldsymbol{\phi}_i^T\mathbf{p}(t)\quad Y_i(t)=\sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}/M_i\quad k=\frac{3EI}{h^3}\quad k=\frac{12EI}{h^3}$$

$$Y_i(t)=\frac{\sin\omega_i t}{M_i\,\omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right]\quad \mathbf{u}(x,t)=\sum\boldsymbol{\phi}_i(x)\,Y_i(t)\quad \ddot{Y}_i(t)+\omega_i^2\,Y_i(t)=0$$

$$\omega^2=\int_o^{\ell}EI(x)[\psi''(x)]^2dx\Big/\int_o^{\ell}m(x)[\psi(x)]^2dx\qquad \sin^2\beta=(1-\cos2\beta)/2$$

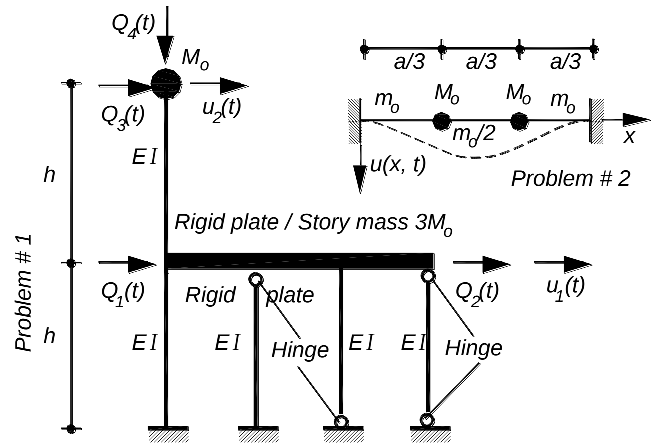
ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / November 26, 2014

Problem # 1:

Consider the system of two degrees-of-freedom shown where the first story is rigid plate having a mass of $3M_o$ and the second story consists of a cantilever column with a lumped mass of M_o at its tip. (a) Evaluate the mass matrix $\mathbf{m}$, the flexibility matrix $\mathbf{d}$, the stiffness matrix $\mathbf{k}$ and the load vector $\mathbf{p}$ of the system. (b). Determine circular frequencies and periods of the free vibration ω_i and T_i in terms of EI, m and h. (c). Obtain corresponding two mode shapes ϕ_i and give their graphical representation ($i=1,2$). (d). Check the orthogonality of the mode shapes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_2$. (e) Evaluate the generalized masses and stiffness $M_i=\boldsymbol{\phi}_i^T\,\mathbf{m}\,\boldsymbol{\phi}_i$ and $K_i=\boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2=K_i/M_i$ ($i=1,2$).

Problem # 2:

Consider the elastic beam shown having a mass per unit length $m_o/2$ at its one-third length and m_o at its two-third length. The beam has two lumped masses M_o additionally. (a) Select one of the displacement functions $\psi_i(x)$ by giving why this function is selected. (b) Obtain first circular frequency of the beam by using Rayleigh ratio.



$\psi_1(x)=x^2$,

$\psi_2(x)=x(a-x)$, $\psi_3(x)=x^2(a-x)^2$, $\psi_4(x)=\sin(\pi x/a)$,

$\psi_5(x)=\sin^2(\pi x/a)$, $\psi_6(x)=\sin(2\pi x/a)$, $\psi_7(x)=\sin^2(2\pi x/a)$

$$\mathbf{m}\,\ddot{\mathbf{u}}(t)+\mathbf{k}\,\mathbf{u}(t)=\mathbf{p}(t)\quad \mathbf{u}(t)=\begin{bmatrix}u_1(t) & u_2(t)\end{bmatrix}^T\quad \mathbf{p}(t)^T=\begin{bmatrix}P_1(t) & P_2(t)\end{bmatrix}\quad \omega_i=2\,\pi/T_i$$

$$(\mathbf{k}-\omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i=\mathbf{0}\quad (\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i=\mathbf{0}\quad \left|\mathbf{k}-\omega_i^2\,\mathbf{m}\right|=0\quad \left|\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m}\right|=0\quad M_i=\boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i=\boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i\quad M_i\,\ddot{Y}_i(t)+K_i\,Y_i(t)=\boldsymbol{\phi}_i^T\mathbf{p}(t)\quad Y_i(t)=\sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}/M_i\quad k=\frac{3EI}{h^3}\quad k=\frac{12EI}{h^3}$$

$$Y_i(t)=\frac{\sin\omega_i t}{M_i\,\omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right]\quad \mathbf{u}(x,t)=\sum\boldsymbol{\phi}_i(x)\,Y_i(t)\quad \ddot{Y}_i(t)+\omega_i^2\,Y_i(t)=0$$

$$\omega^2=\int_o^{\ell}EI(x)[\psi''(x)]^2dx\Big/\int_o^{\ell}m(x)[\psi(x)]^2dx\qquad \sin^2\beta=(1-\cos2\beta)/2$$