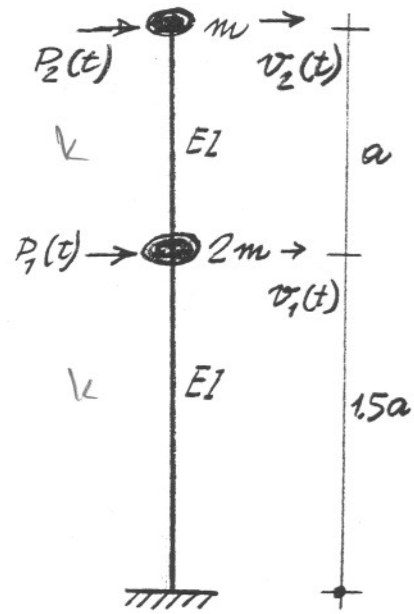
ADVANCED DYNAMICS OF STRUCTURES **Midterm Exam** **November 07, 2005**

1. Obtain the mass matrix **m** and the flexibility matrix **d** of the two-story shear frame shown having two-degree-of-freedom.
2. Evaluate the stiffness matrix $\mathbf{k} = \mathbf{d}^{-1}$. Write down equation of motion including the external forces $P_1(t)$ and $P_2(t)$.
3. Determine the two circular frequencies $\omega_1 < \omega_2$, the two periods of the free vibration and $T_1 > T_2$ and the corresponding mode shapes ϕ_1, and ϕ_2. Give their graphical representations.
4. Check the orthogonality of the modes with respect to the mass matrix $\phi_1^T \mathbf{m}\, \phi_2$ and the stiffness matrix $\phi_1^T \mathbf{k}\, \phi_2$.
5. Evaluate the generalized masses and stiffness M_1, M_2 and K_1, K_2 and assess that $\omega_i^{\,2} = K_i / M_i$.
6. Write down the discretized version of the equations of motion as $M_i\, \ddot{Y}_i + K_i\, Y_i = \phi_i^T\ p_i$
7. Assuming that the system is under the impulsive load of P_2 ad $I = \int_0^{t_o} P_2(\tau)\, d\tau = P_o\, t_o$ and $P_1(t) = 0$ and assuming that the system starts from the rest, i.e., $v(t=0) = \dot{v}(t=0) = 0$ and $t_o << T_2 < T_1$, obtain $Y_i(t)$ and $v_i(t)$.
8. For a numerical evaluation, assume that $t_o = T_2 / 10$, $P_o = 3mg$, $T_1 =$ *1 second*, calculate v_1 $(t = 0.25\ second)$ and v_2 $(t = 0.25\ second)$.

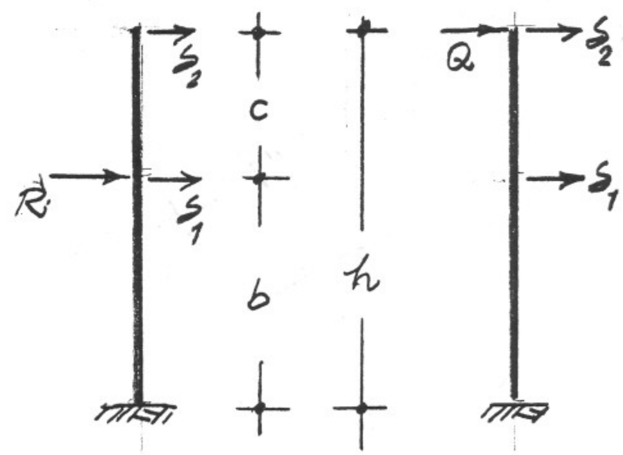


$$\mathbf{m}\, \ddot{\mathbf{v}}(t) + \mathbf{k}\, \mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = [v_1(t) \quad v_2(t)]^T \qquad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)] \qquad (\mathbf{k} - \omega_i^2\, \mathbf{m})\, \phi_i = 0$$

$$(\mathbf{I} - \omega_i^2\, \mathbf{d}\, \mathbf{m})\, \phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\, \mathbf{m}\right| = 0 \qquad \left|\mathbf{I} - \omega_i^2\, \mathbf{d}\, \mathbf{m}\right| = 0 \qquad \omega_i = 2\pi / T_i \qquad M_i = \phi_i^T \mathbf{m}\, \phi_i \qquad K_i = \phi_i^T \mathbf{k}\, \phi_i$$

$$M_i\, \ddot{Y}_i(t) + K_i\, Y_i(t) = \phi_i^T \mathbf{p}(t) \qquad v(t) = \sum_{i=1}^{2} Y_i(t)\, \phi_i \qquad Y_i(t) = \sum_{i=1}^{2} \phi_i^{\,T}\, \mathbf{m}\, \mathbf{v} / M_i$$

$$Y_i(t) = \frac{\sin \omega_i t}{M_i\, \omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\, d\tau\right]$$



$$\delta_1 = \frac{Q\, b^3}{3\, EI} + \frac{Q\, c\, b^2}{2\, EI} \qquad \delta_2 = \frac{Q\, h^3}{3\, EI}$$

$$\delta_1 = \frac{R\, b^3}{3\, EI} \qquad \delta_2 = \frac{R\, b^3}{3\, EI} + \frac{R\, c\, b^2}{2\, EI}$$