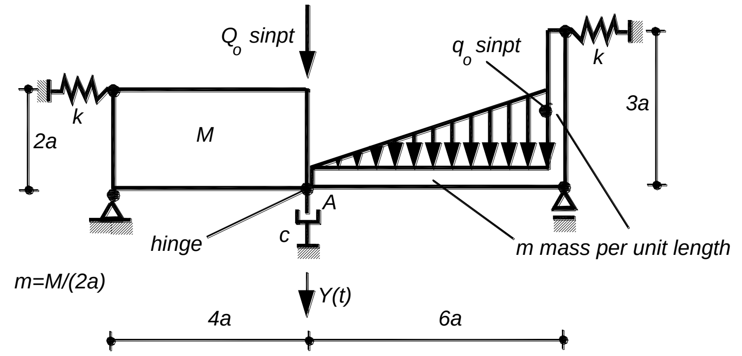
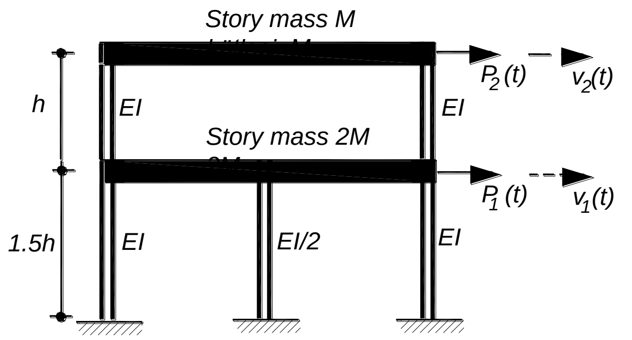
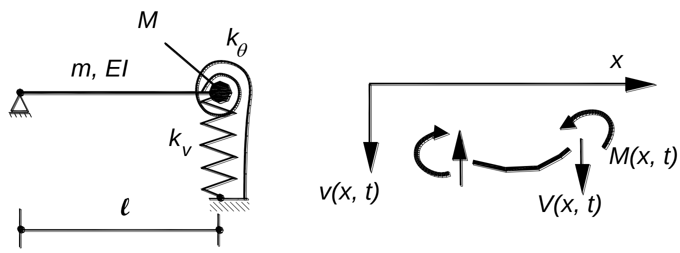
1. For the rigid--body assemblage shown,
   a. Set up the equation of motion for the generalized displacement $Y(t)$ of the point A by using the principle of the virtual work.
   b. By assuming $m = M/(2a)$ determine the period of the system as $T = 2\pi/\omega = \alpha\sqrt{ma/k}$ and evaluate α.
   c. By assuming $q_o = Q_o/(5a)$ and $p = 0.9\omega = 0.9 \times 2\pi/T$ determine $Y(t)$ the particular (steady-state) solution of the equation of motion.



2. Consider the system of two-degree-of freedom which behaves as a shear frame.
   a. Write down equation of motion by including the external forces $P_1(t)$ and $P_2(t)$. Obtain the mass matrix **m** and the stiffness **k** of the system.
   b. Determine the two circular frequencies $\omega_1 < \omega_2$, the two periods of the free vibration $T_1 > T_2$ and the corresponding mode shapes $\boldsymbol{\phi}_1$, and $\boldsymbol{\phi}_2$. Give their graphical representations.
   c. Check the orthogonality of the modes with respect to the mass matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$ and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$.
   d. Evaluate the generalized masses and stiffness $M_1\ \ M_2$ and $K_1\ \ K_2$ and assess that $\omega_i^2 = K_i / M_i$.
   e. Determine the effective modal masses M_1^*, M_2^* and asses that $M_1^* + M_2^* = 3\,M$





3. Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has a lumped mass of M at the left end.
   a. Write down the boundary conditions for the free vibration of the beam.
   b. By assuming $M = m\,\ell$, $k_v = EI/\ell^3$ and $k_\theta = EI/\ell$ obtain the frequency determinant.

$\omega^2 = k/m \qquad \omega = 2\pi/T \qquad I = \int_0^{t_1} p(t)\,dt \qquad v(t) = \dfrac{I}{m\omega}\sin\omega t \qquad v(t) = \dfrac{1}{m\omega}\int_0^t p(\tau)\sin\omega(t-\tau)\,d\tau \qquad I_\theta = \dfrac{M}{12}(a^2+b^2)$

$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = [v_1(t)\quad v_2(t)]^T \qquad \mathbf{p}(t)^T = [P_1(t)\quad P_2(t)] \qquad (\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0$

$\left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \qquad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \qquad \omega_i = 2\pi/T_i \qquad M_i = \phi_i^T\mathbf{m}\,\phi_i \qquad K_i = \phi_i^T\mathbf{k}\,\phi_i \qquad v(t) = \sum_{i=1}^{2} Y_i(t)\,\phi_i$

$M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\mathbf{p}(t) \qquad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \qquad Y_i(t) = \sum_{i=1}^{2}\phi_i^T\,\mathbf{m}\,\mathbf{v}/M_i \qquad Y_i(t) = \dfrac{\sin\omega_i t}{M_i\,\omega_i}\left[\phi_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right]$

$L_i = \phi_i^T\,\mathrm{m}\,1 \qquad \Gamma_i = L_i / M_\mathrm{i} \qquad M_i^* = \Gamma_i\,L_i \qquad 1^\mathrm{T} = [1\quad 1] \qquad v(x,t) = \sum\phi_i(x)\,Y_i(t)$

$\phi(x) = A_1\sin ax + A_2\cos ax + A_3\sinh ax + A_4\cosh ax \qquad a^4 = \dfrac{m\,\omega^2}{EI} \qquad V(x,t) = -EI\,\dfrac{\partial^3 v}{\partial x^3} \qquad M(x,t) = -EI\,\dfrac{\partial^2 v}{\partial x^2}$

$k = 3EI/h^3 \qquad k = 12EI/h^3$

H.Boduroğlu /Z.Celep