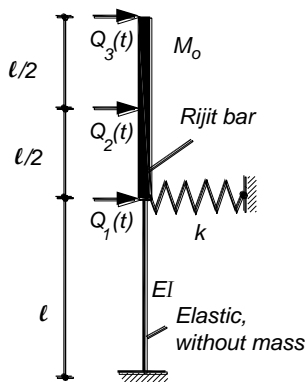


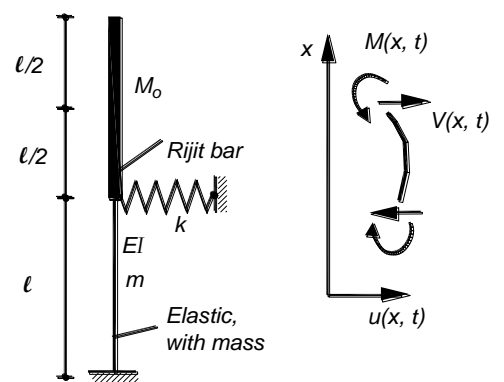
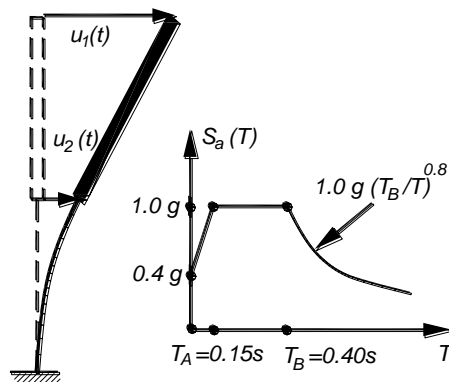
**Problem # 1**

Consider the system of two degrees-of-freedom shown:

- Write down the equations of motion of the system by including the external loads.
- Determine the two circular frequencies and the periods of the free vibration  $\omega_i$  and  $T_i$  and the corresponding mode shapes  $\phi_i$ . Give their graphical representation ( $i = 1, 2$ ),
- Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix  $\phi_1^T \mathbf{m} \phi_2$ , and  $\phi_1^T \mathbf{k} \phi_2$ ,
- Evaluate the generalized masses and stiffness  $M_i = \phi_i^T \mathbf{m} \phi_i$ , and  $K_i = \phi_i^T \mathbf{k} \phi_i$ , and assess  $\omega_i^2 = K_i / M_i$  for  $i = 1, 2$ ,
- The height of the column  $\ell = 3\text{meter}$  and their cross section of  $b/h = 0.25\text{m} \times 0.50\text{m}$ , the first period of the system is  $T_1 = 0.20\text{s}$  and  $E = 30\text{GPa}$ . Find the numerical values the parameter  $M_o$  and the second period  $T_2$  of the system.
- Determine the effective modal masses  $M_1^*$  and  $M_2^*$  and assess that  $M_1^* + M_2^* = 3M_o$
- Evaluate the base shear forces  $V_{b1}$  and  $V_{b2}$  and the overturning moments  $M_{b1}$  and  $M_{b2}$  corresponding to the two mode shapes. Obtain the base shear force  $V_b$  and the overturning moment  $M_b$  by using the SRSS combination rule.



Problem # 1



Problem # 2

**Problem # 2**

Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant by assuming  $M_o = m a$ ,

$$k = EI / \ell^3 \text{ in terms of } \beta \text{ where } \beta^4 = \frac{m \ell^4 \omega^2}{EI}$$

$$\mathbf{m} \ddot{\mathbf{u}}(t) + \mathbf{k} \mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = [u_1(t) \quad u_2(t)]^T \quad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)] \quad (\mathbf{k} - \omega_i^2 \mathbf{m}) \phi_i = 0 \quad \omega_i = 2 \pi / T_i$$

$$(\mathbf{I} - \omega_i^2 \mathbf{d} \mathbf{m}) \phi_i = 0 \quad \left| \mathbf{k} - \omega_i^2 \mathbf{m} \right| = 0 \quad K_i = \phi_i^T \mathbf{k} \phi_i \quad M_i \ddot{Y}_i(t) + K_i Y_i(t) = \phi_i^T \mathbf{p}(t) \quad \mathbf{u}(x, t) = \sum \phi_i(x) Y_i(t)$$

$$\ddot{Y}_i(t) + \omega_i^2 Y_i(t) = 0 \quad Y_i(t) = \frac{\sin \omega_i t}{M_i \omega_i} \left[ \phi_i^T \int_0^{t_0} \mathbf{p}(\tau) d\tau \right] \quad Y_i(t) = \phi_i^T \mathbf{m} \mathbf{v} / M_i$$

$$M_i = \phi_i^T \mathbf{m} \phi_i \quad I_\theta = \frac{M}{12} (a^2 + b^2) \quad M(x, t) = -EI \frac{\partial^2 u}{\partial x^2} \quad V(x, t) = -EI \frac{\partial^3 u}{\partial x^3} \quad a^4 = \frac{m \omega^2}{EI}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \quad L_i = \phi_i^T \mathbf{m} \mathbf{1} \quad \Gamma_i = L_i / M_i \quad M_i^* = \Gamma_i L_i$$

$$I_\theta = \frac{M}{12} (a^2 + b^2) \quad V_{bj} = M_j^* A_{jmax} \quad f_{ij} = V_{bj} \frac{m_i \phi_{ij}}{\sum_{l=1}^n m_l \phi_{lj}} \quad M_j^* = \frac{\left( \sum_{i=1}^n m_i \phi_{ij} \right)^2}{\sum_{i=1}^n m_i \phi_{ij}^2}$$