ADVANCED DYNAMICS OF STRUCTURES Final Exam January 10, 2011 / *HBoduroğlu/ZCelep*

Problem # 1

Consider the system of two degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the external loads.

b. Determine the two circular frequencies and the periods of the free vibration ω_i and T_i and the corresponding mode shapes $\boldsymbol{\phi}_i$. Give their graphical representation ($i = 1, 2$),

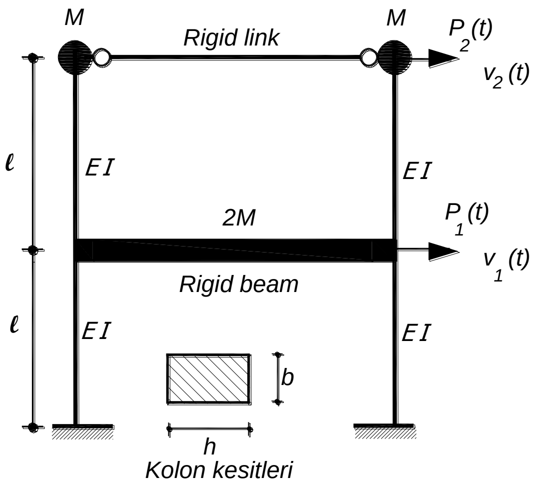
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$,

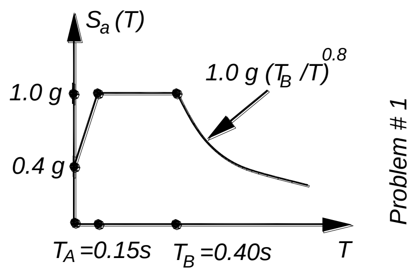
d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess $\omega_i^{\,2} = K_i / M_i$ for $i = 1, 2$,

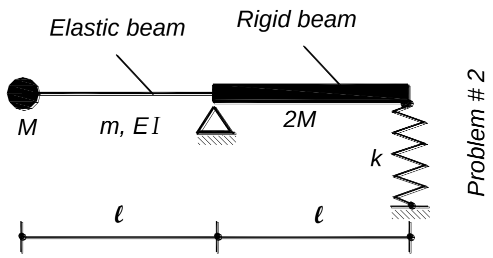
e. The heights of the stories are $\ell = 3meter$, the columns have cross section of $b / h = 0.30m \times 0.60m$, the first period of the system is $T_1 = 0.25s$ and $E = 30GPa$. Find the numerical values the parameter M and the second period T_2 of the system.

f. Determine the effective modal masses M_1^* and M_2^* and assess that $M_1^* + M_2^* = 4M$

g. Evaluate the base shear forces V_{b1} and V_{b2} corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases and the story shear forces by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.







Problem # 2

Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant by assuming $M = m\,\ell$, $k = E\,I/\ell^3$ in terms of β where $\beta^4 = \dfrac{m\,\ell^4\,\omega^2}{EI}$

$$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = \left[v_1(t) \quad v_2(t)\right]^T \qquad \mathbf{p}(t)^T = \left[P_1(t) \quad P_2(t)\right]$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad \omega_i = 2\,\pi / T_i \qquad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0$$

$$K_i = \phi_i^T \mathbf{k}\,\phi_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T \mathbf{p}(t) \quad \mathbf{v}(x,t) = \sum \phi_i(x)\,Y_i(t)$$

$$\ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \quad Y_i(t) = \frac{\sin \omega_i t}{M_i\,\omega_i}\left[\varphi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad Y_i(t) = \phi_i^T\,\mathbf{m}\,\mathbf{v} / M_i$$

$$M_i = \phi_i^T \mathbf{m}\,\phi_i \qquad I_\theta = \frac{M}{12}(a^2 + b^2) \qquad M(x,t) = -EI\,\frac{\partial^2 v}{\partial x^2} \qquad V(x,t) = -EI\,\frac{\partial^3 v}{\partial x^3} \qquad a^4 = \frac{m\,\omega^2}{EI}$$

$$\varphi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad L_i = \phi_i^T \mathbf{m}\,\mathbf{1} \qquad \Gamma_i = L_i \,/\, M_i \qquad M_i^* = \Gamma_i\,L_i$$

$$I_\theta = \frac{M}{12}(a^2 + b^2) \qquad V_{b\,j} = M_j^*\,A_{j\,max} \qquad f_{ij} = V_{b\,j}\,\frac{m_i\,\phi_{ij}}{\sum_{l=1}^{n} m_l\,\phi_{lj}} \qquad M_j^* = \frac{\left(\sum_{i=1}^{n} m_i\,\varphi_{ij}\right)^2}{\sum_{i=1}^{n} m_i\,\varphi_{ij}^2}$$