ADVANCED DYNAMICS OF STRUCTURES / Final exam / January 14, 2010

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Problem # 1:

Consider the system of two degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the ground motion $v_g(t)$ and evaluate the mass matrix **m**, the rigidity matrix **k,** and the flexibility matrix **k = d**$^{-1}$,

b. Determine the two circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, M and ℓ. Obtain the corresponding mode shapes $\boldsymbol{\phi}_i$ and give their graphical representations ($i = 1, 2$),

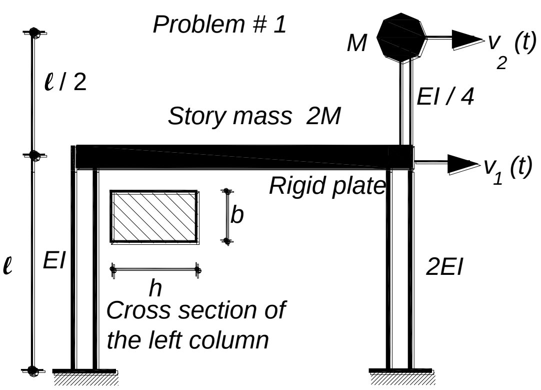
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$ and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$.

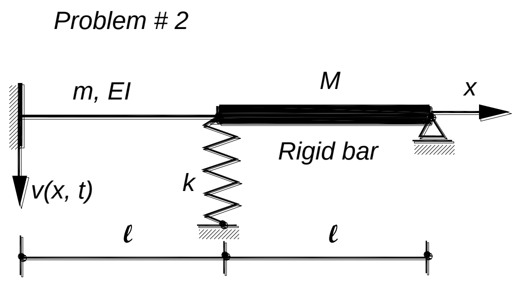
d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess $\omega_i^{\,2} = K_i / M_i$. ($i = 1, 2$),

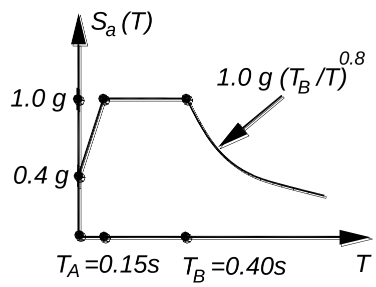
e. The story height is $\ell = 3meter$, the left hand side column has a cross section of $b / h = 0.30m \times 0.60m$, the first period of the system is $T_1 = 0.25s$ and $E = 30GPa$. Find the numerical values the parameter M and the second period of the system.

f. Determine the effective modal masses M_1^* and M_2^* and assess that $M_1^* + M_2^* = 3M$

g. Evaluate the base shear forces V_{b1} and V_{b2} corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases, the story shear forces and the story displacements by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.







Problem # 2:

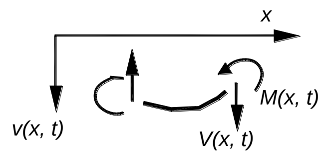
Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has a clamped support at the left end. It is connected to a rigid bar of a mass M and to a spring k.

a. Write down the boundary conditions for the free vibration of the beam.

b. By assuming $M = m\,\ell$ and $k = E\,I / \ell^3$ obtain the frequency determinant in terms of $\beta_i^4 = \dfrac{m\,\ell^4\,\omega_i^2}{EI}$.

$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = [v_1(t) \quad v_2(t)]^T \qquad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)]$

$(\mathbf{k} - \omega_i^2\ \mathbf{m})\,\phi_i = 0 \qquad \omega_i = 2\,\pi / T_i \qquad (\mathbf{I} - \omega_i^2\ \mathbf{d}\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\ \mathbf{m}\right| = 0$



$K_i = \phi_i^T \mathbf{k}\,\phi_i \quad M_i = \phi_i^T \mathbf{m}\,\phi_i \quad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T \mathbf{p}(t) \qquad v(t) = \sum_{i=1}^{2} Y_i(t)\,\phi_i$

$Y_i(t) = \phi_i^T\ \mathbf{m}\,\mathbf{v} / M_i \qquad Y_i(t) = \dfrac{\sin\omega_i t}{M_i\ \omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad L_i = \phi_i^T \mathrm{m}\,1 \qquad \Gamma_i = L_i / M_i \qquad M_i^* = \Gamma_i\ L_i$

$I_\theta = \dfrac{M}{12}(a^2 + b^2) \qquad M(x,t) = -EI\dfrac{\partial^2 v}{\partial x^2} \qquad V(x,t) = -EI\dfrac{\partial^3 v}{\partial x^3} \qquad a^4 = \dfrac{m\,\omega^2}{EI} \qquad v(x,t) = \sum \phi_i(x)\,Y_i(t)$

$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad \ddot{Y}_i(t) + \omega_i^2\ Y_i(t) = 0$

$f_{ij} = V_{b\,j}\dfrac{m_i\ \phi_{ij}}{\sum_{l=1}^{n} m_l\ \phi_{lj}} \qquad M_j^* = \dfrac{\left(\sum_{i=1}^{n} m_i\ \phi_{ij}\right)^2}{\sum_{i=1}^{n} m_i\ \phi_{ij}^2} \qquad M_i^* = \Gamma_i\ L_i \qquad V_{b\,j} = M_j^*\ A_{j\,max}$