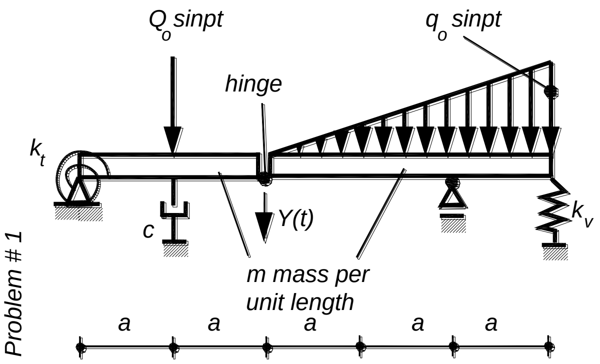
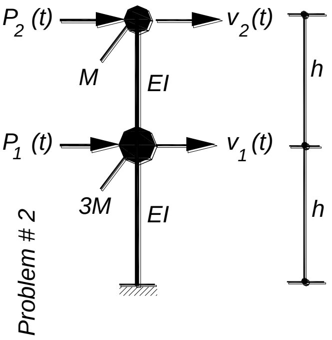
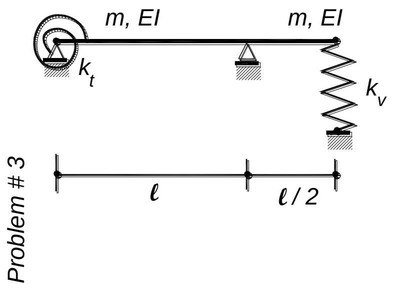
ADVANCED DYNAMICS OF STRUCTURES Final Exam January 12, 2009 / *HBoduroğlu/ZCelep*







Problem # 1

For the rigid-body assemblage shown set up the equation of motion in terms of the vertical displacement $Y(t)$ of the hinge by using the principle of the virtual work. By assuming $k_t = 2\,k_v\,a^2$, determine the period of the system as $T = \alpha\sqrt{m\,a/k_v}$ and evaluate α.

Problem # 2

a. Write down equation of motion of the column having two-degree-of-freedom including the external forces $P_1(t)$ and $P_2(t)$.

b. Obtain the mass matrix **m**, the stiffness matrix **k** and the flexibility matrix $\mathbf{d} = \mathbf{k}^{-1}$.

c. Determine the two circular frequencies $\omega_1 < \omega_2$, the two periods of the free vibration $T_i = \alpha_i\sqrt{M\,h^3/(EI)}$ $T_1 > T_2$ including α_1 and the corresponding mode shapes $\boldsymbol{\phi}_1$, and $\boldsymbol{\phi}_2$. Give their graphical representations.

d. Check the orthogonality of the modes with respect to the mass matrix $\boldsymbol{\phi}_1^{\mathbf{T}}\mathbf{m}\,\boldsymbol{\phi}_2$ and the stiffness matrix $\boldsymbol{\phi}_1^{\mathbf{T}}\mathbf{k}\,\boldsymbol{\phi}_2$.

e. Evaluate the generalized masses and stiffness M_1, M_2 and K_1, K_2 and assess that $\omega_i^2 = K_i/M_i$.

f. Write down the discretized version of the equations of motion as $M_i\,\ddot{Y}_i + K_i\,Y_i = \boldsymbol{\phi}_1^{\mathbf{T}}\,\mathbf{p}$

g. Assuming that the system is under the impulsive load of P_2 ad $I = \int_0^{t_o} P_2(\tau)\,d\tau = P_o\,t_o$ and $P_1(t) = 0$ and assuming that the system starts from the rest, i.e., $v(t=0) = \dot{v}(t=0) = 0$ and $t_o << T_2 < T_1$, obtain $Y_i(t)$ and $v_i(t)$.

Problem # 3

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. Write down the boundary conditions for the free vibration of the beam.

$$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = \begin{bmatrix} v_1(t) & v_2(t) \end{bmatrix}^T \qquad \mathbf{p}(t)^T = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix}$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad \omega_i = 2\,\pi/T_i \qquad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0$$

$$K_i = \phi_i^T\,\mathbf{k}\,\phi_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\,\mathbf{p}(t) \qquad v(t) = \sum_{i=1}^{2} Y_i(t)\,\phi_i$$

$$Y_i(t) = \phi_i^T\,\mathbf{m}\,\mathbf{v}/M_i \qquad Y_i(t) = \frac{\sin\omega_i t}{M_i\,\omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad M_i = \phi_i^T\,\mathbf{m}\,\phi_i$$

$$I_\theta = \frac{M}{12}(a^2 + b^2) \qquad v_1 = \frac{Q\,b^3}{3\,EI} + \frac{Q\,c\,b^2}{2\,EI} \qquad v_2 = \frac{Q\,(c+b)^3}{3\,EI} \qquad v_1 = \frac{R\,b^3}{3\,EI}$$

$$v_2 = \frac{R\,b^3}{3\,EI} + \frac{R\,c\,b^2}{2\,EI} \qquad M(x,t) = -EI\,\frac{\partial^2 v}{\partial x^2} \qquad V(x,t) = -EI\,\frac{\partial^3 v}{\partial x^3} \qquad a^4 = \frac{m\,\omega^2}{EI}$$

$$v(x,t) = \sum \phi_i(x)\,Y_i(t) \qquad \phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0$$