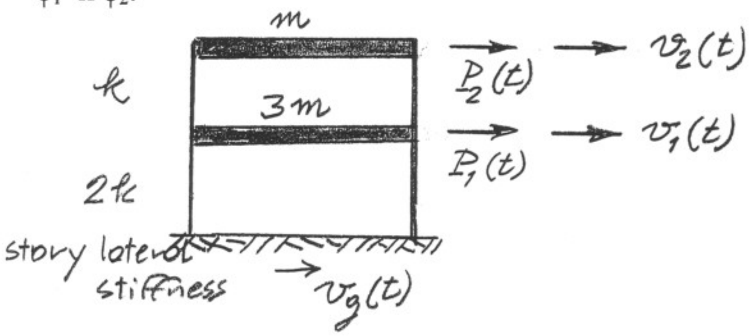
ADVANCED DYNAMICS OF STRUCTURES **Final Exam** **January 16, 2007**

(1) Consider the system of two-degrees-of-freedom shown:

a Write down equations of motion of the system by including the ground motion $v_g(t)$ and the external loads $P_1(t)$ and $P_2(t)$. Obtain the mass matrix **m** and the rigidity matrix **k.**

b Determine the two circular frequencies and the two periods of the free vibration $\omega_1 < \omega_2$, and $T_1 > T_2$ and the corresponding mode shapes ϕ_1 and ϕ_2. Give their graphical representations.

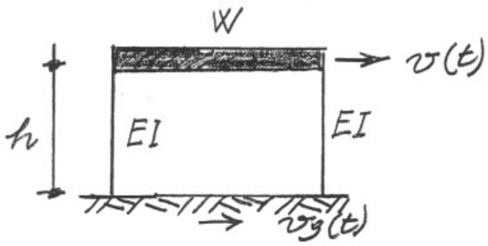
c Check the orthogonality of the modes with respect to the mass matrix $\phi_1^T \mathbf{m}\, \phi_2$ and the stiffness matrix $\phi_1^T \mathbf{k}\, \phi_2$.

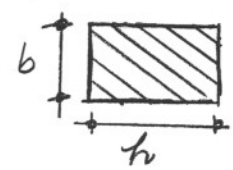


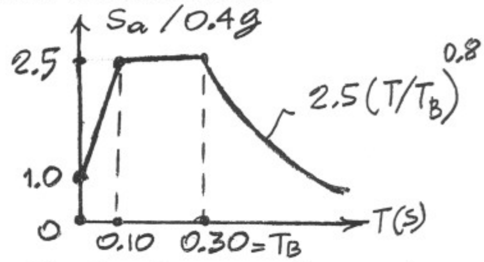
(2) Consider the system of single-degrees-of-freedom shown:

a. Write down equation of motion of the system by including the ground motion $v_g(t)$ and obtain the period of the system.

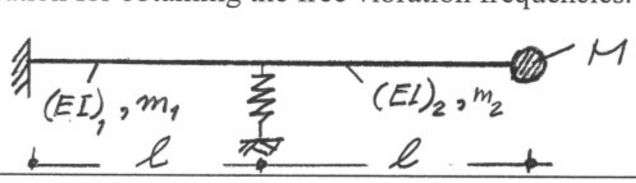
b. Assuming $h = 3m$ and the cross section of the columns $b = 0.30m$, $h = 0.40m$ and $E = 30\ GPa$, $T = 0.2s$ obtain the weight of the system. By using the spectral curve given, obtain the base shear force V_b and the corresponding lateral deformation of the system. Determine the shear forces at each column.







(3) Consider the cantilever beam having a lumped mass and a lateral spring at the tip where m_1 and m_2 are the masses per unit length and $(EI)_1$ and $(EI)_2$ the bending rigidities of the cross sections, k the constant of the spring and M the lumped mass at the tip. Write down the boundary conditions of the beam that experiences free vibration for obtaining the free vibration frequencies.



$$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = [v_1(t),\ v_2(t)]^T \qquad \mathbf{p}(t)^T = [P_1(t),\ P_2(t)]$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \qquad \omega_i = 2\pi / T_i$$

$$M_i = \phi_i^T\,\mathbf{m}\,\phi_i \qquad K_i = \phi_i^T\,\mathbf{k}\,\phi_i \qquad v(x,t) = \phi(x)\,Y(t)$$

$$\ddot{Y}(t) + \omega^2 Y(t) = 0 \qquad M(x,t) = -EI\,\partial^2 v/\partial x^2 \qquad V(x,t) = -EI\,\partial^3 v/\partial x^3$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad a^4 = m\omega^2 / EI$$

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