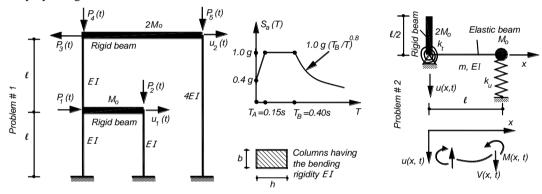
## ADVANCED DYNAMICS OF STRUCTURES / Final Exam / January 15, 2014

## Problem # 1:

- a. Consider the system of two degree-of-freedom shown where the first and the second stories are rigid plates having a mass of  $3M_o$ and  $M_o$ , respectively. (a) Write down equations of motion by considering the free body diagram of the two story masses separately. (b) Evaluate the mass matrix  $\mathbf{m}$ , and the rigidity matrix  $\mathbf{k}$  and the load vector  $\mathbf{p}$ . (c) Determine the circular frequencies  $\omega_i$  and the periods  $T_i$  of the free vibration in terms of EI,  $M_o$  and  $\ell$ . (d) Obtain the corresponding two mode shapes  $\phi_i$  and give their graphical representation (i = 1, 2). (e) Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix  $\phi_1^T \mathbf{m} \phi_2$ , and  $\phi_1^T \mathbf{k} \phi_2$ . (f). Evaluate the generalized masses and stiffness  $M_i = \phi_i^T \mathbf{m} \phi_i$  and  $K_i = \phi_i^T \mathbf{k} \phi_i$  and assess the relationship  $\omega_i^2 = K_i / M_i$  (i = 1, 2). Determine the effective modal masses  $M_1^*$  and  $M_2^*$ , and assess  $M_1^* + M_2^* = 3M_o$
- b. The heights of the stories are  $\ell = 3meter$ , the columns have cross section of b/h = 0.25m/0.50m, the weight  $M_og = 400kN$  and E = 30GPa. Find the first and second periods  $T_1$  and  $T_2$  of the system.
- c. Evaluate the base shear forces  $V_{b1}$  and  $V_{b2}$  corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases and the story shear forces by using the acceleration spectrum given. Obtain the story forces  $f_{21}$  and  $f_{22}$  due to the base shear forces  $V_{b1}$  and  $V_{b2}$  and the bending moments at the columns of the second story. Obtain the shear force at the second story by using the SRSS combination rule.



## Problem # 2:

Consider an elastic beam having a cross sectional bending rigidity having *EI*, a mass per unit length *m* and a length  $\ell$ . The left end of the elastic beam is simply supported having a rotational spring with a spring constant  $k_t$  and a vertical rigid beam with a mass  $2M_o$ ; its right end has a mass  $M_o$  and it is connected to a lateral spring having a spring constant  $k_u$ . Write down the boundary conditions for the free vibration of the system. Obtain the frequency determinant in terms of  $\beta^4 = (a\ell)^4 = m\ell^4\omega^2/(EI)$  by assuming  $M_o = 2m\ell$ ,  $k_u = 2EI/\ell^3$  and  $k_t = 2EI/\ell$ .

Problem # 3: Indicate if the following statements are true or false such as (a) T or (a) F:

- (a) Application of base isolators to a building often leads to a reduction in the design total base shear force.
- (b) There are no base isolated buildings in Turkey.
- (c) The gas and water pipeline connections for base isolated buildings must be specially built to accommodate for the expected displacements.
- (d) Turkish Seismic Code has a specific chapter for design of base isolated buildings.
- (e) A base isolated building is expected to sustain <u>less damage</u> compared to conventional building, when subjected to the design ground motion intensity shaking.

$$\mathbf{m} \ddot{\mathbf{u}}(t) + \mathbf{k} \mathbf{u}(t) = \mathbf{p}(t) \ \mathbf{u}(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \ \mathbf{p}(t)^T = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \quad \omega_i = 2 \pi / T_i$$

$$(\mathbf{k} - \omega_i^2 \ \mathbf{m}) \ \mathbf{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2 \ \mathbf{d} \ \mathbf{m}) \ \mathbf{\phi}_i = 0 \quad \left| \mathbf{k} - \omega_i^2 \ \mathbf{m} \right| = 0 \quad \left| \mathbf{I} - \omega_i^2 \ \mathbf{d} \ \mathbf{m} \right| = 0 \quad M_i = \mathbf{\phi}_i^T \ \mathbf{m} \ \mathbf{\phi}_i$$

$$K_i = \mathbf{\phi}_i^T \mathbf{k} \ \mathbf{\phi}_i \quad M_i \ \ddot{Y}_i(t) + K_i \ Y_i(t) = \mathbf{\phi}_i^T \ \mathbf{p}(t) \quad Y_i(t) = \sum_{i=1}^2 \mathbf{\phi}_i^T \ \mathbf{m} \ \mathbf{v} / M_i \quad k = \frac{3EI}{h^3} \quad k = \frac{12EI}{h^3} \quad k = \frac{12EI}{h^3}$$

$$Y_i(t) = \frac{\sin \omega_i t}{M_i \ \omega_i} \left[ \mathbf{\phi}_i^T \int_o^{t_0} \mathbf{p}(\tau) \ d\tau \right] \quad L_i = \mathbf{\phi}_i^T \ \mathbf{m} \ \mathbf{1} \quad \Gamma_i = L_i / M_i \quad M_i^* = \Gamma_i \ L_i \quad \mathbf{1} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \quad V_{bj} = M_j^* \ S_a(T_j)$$

$$u(x,t) = \sum \mathbf{\phi}_i(x) \ Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2 \ Y_i(t) = 0 \quad M(x,t) = -EI \frac{\partial^2 u}{\partial x^2} \quad V(x,t) = -EI \frac{\partial^3 u}{\partial x^3} \quad f_{nj} = V_{bj} \frac{m_n \ \phi_{nj}}{\sum_k m_k \ \phi_{kj}}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \quad a^4 = \frac{m \ \omega^2}{EI}$$