ADVANCED DYNAMICS OF STRUCTURES Final Exam January 11, 2012 / *ZCelep*

Problem # 1

Consider the system of two degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the external loads. Develop the stiffness matrix $\mathbf{k}$ and the load vector $\mathbf{p}(t)$

b. Determine the two circular frequencies and the periods of the free vibration ω_i and T_i and the corresponding mode shapes ϕ_i. Give their graphical representation ($i = 1, 2$),

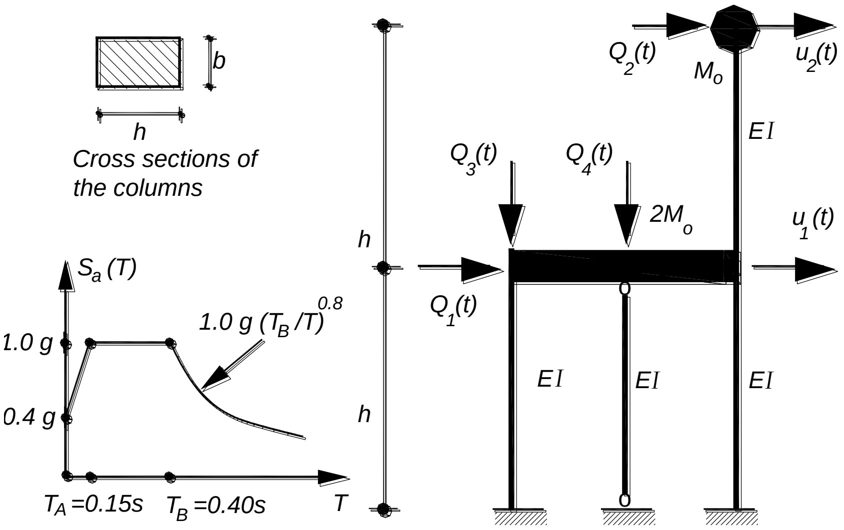
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\phi_1^T \mathbf{m}\, \phi_2$, and $\phi_1^T \mathbf{k}\, \phi_2$,

d. Evaluate the generalized masses and stiffness $M_i = \phi_i^T \mathbf{m}\, \phi_i$, and $K_i = \phi_i^T \mathbf{k}\, \phi_i$, and assess $\omega_i^{\,2} = K_i / M_i$ for $i = 1, 2$,

e. The heights of the stories are $\ell = 3 meter$, the columns have cross section of $b / h = 0.25m \times 0.50m$, the first period of the system is $T_1 = 0.20s$ and $E = 30GPa$. Find the numerical values the parameter M and the second period T_2 of the system.

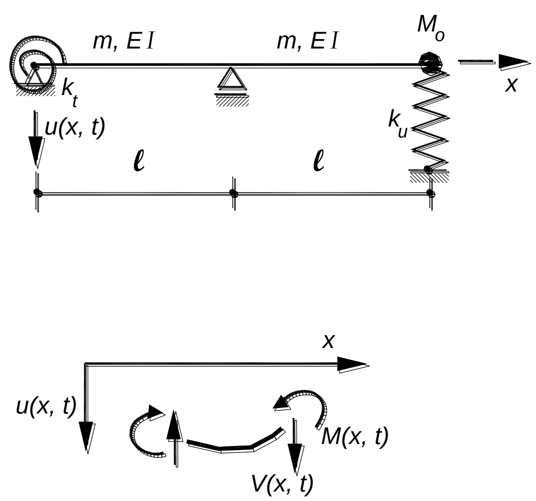
f. Determine the effective modal masses M_1^* and M_2^* and assess that $M_1^* + M_2^* = 3M_o$

g. Evaluate the base shear forces V_{b1} and V_{b2} and the overturning moments M_{b1} and M_{b2} corresponding to the two mode shapes. Obtain the base shear force V_b and the overturning moment M_b by using the SRSS combination rule.



Problem # 1

Problem # 2



Problem # 2

Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant by assuming $M_O = m\,\ell$, $k_u = E\,I / \ell^3$ and $k_t = E\,I / \ell$ in terms of β where $\beta^4 = \dfrac{m\,\ell^4\,\omega^2}{EI}$

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \qquad \mathbf{u}(t) = [u_1(t) \quad u_2(t)]^T \qquad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)] \qquad (\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad \omega_i = 2\pi / T_i$$

$$(\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \qquad K_i = \phi_i^T\,\mathbf{k}\,\phi_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\,\mathbf{p}(t) \qquad \mathbf{u}(x,t) = \sum \phi_i(x)\,Y_i(t)$$

$$\ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \quad Y_i(t) = \frac{\sin\omega_i t}{M_i\,\omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad Y_i(t) = \phi_i^T\,\mathbf{m}\,\mathbf{v} / M_i$$

$$M_i = \phi_i^T\,\mathbf{m}\,\phi_i \qquad I_\theta = \frac{M}{12}(a^2 + b^2) \qquad M(x,t) = -EI\,\frac{\partial^2 u}{\partial x^2} \qquad V(x,t) = -EI\,\frac{\partial^3 u}{\partial x^3} \qquad a^4 = \frac{m\,\omega^2}{EI}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad L_i = \phi_i^T\,\mathbf{m}\,\mathbf{1} \qquad \Gamma_i = L_i / M_i \qquad M_i^* = \Gamma_i\,L_i$$

$$I_\theta = \frac{M}{12}(a^2 + b^2) \qquad V_{bj} = M_j^*\,A_{j\,max} \qquad f_{ij} = V_{bj}\,\frac{m_i\,\phi_{ij}}{\sum_{l=1}^{n} m_l\,\phi_{lj}} \qquad M_j^* = \frac{\left(\sum_{i=1}^{n} m_i\,\phi_{ij}\right)^2}{\sum_{i=1}^{n} m_i\,\phi_{ij}^2}$$