**ADVANCED DYNAMICS OF STRUCTURES / Final Exam / December 22, 2014**

***Question # 1:***

Consider the system of two degrees-of-freedom shown shown:

a. Write down the equations of motion of the system by considering the freebody diagram of the two story masses. Evaluate the mass matrix $\mathbf{m}$, the stiffness matrix $\mathbf{k}$ **.**

b. Determine the two circular frequencies and periods of the free vibration $\omega_i$ and $T_i$ in terms of $M_o$, $EI$ and $\ell$. Obtain the corresponding mode shapes $\boldsymbol{\phi}_i$ and give their graphical representations (*i =1, 2*).

a. Determine the effective modal masses $M_1^*$ and $M_2^*$, and assess the identity $M_1^* + M_2^* = 3M_o$

b. Assuming the heights of the stories are $\ell = 3meter$, the columns with a cross section of $b/h = 0.40m/0.60m$ and the first period of the system $T_1 = 0.15s$ and the modulus elasticity $E = 30GPa$, find the numerical values of the mass $M_o$ and the second period $T_2$ of the system.

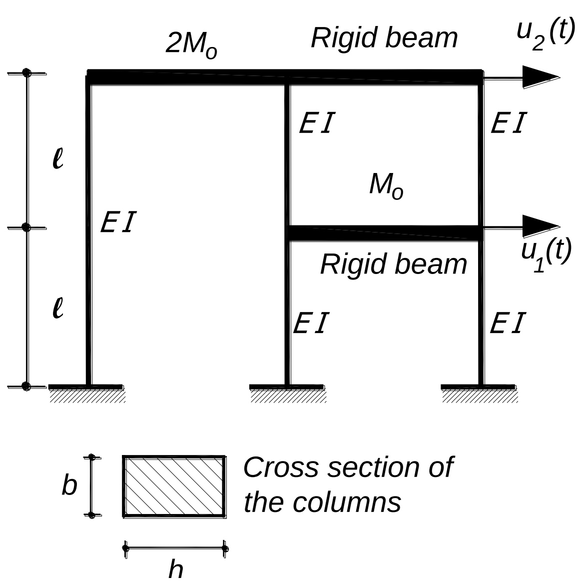
c. Evaluate the base shear forces $V_{b1}$, $V_{b2}$ corresponding to the two mode shapes by using the acceleration spectrum $S_a(T)$ given. Obtain the base shear force and their combined value $V_b$ by using the SRSS combination rule.

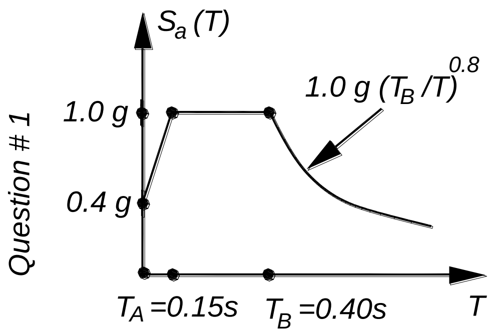
***Question # 2:***

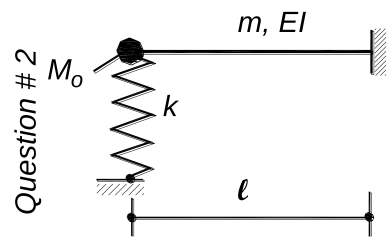
Consider a beam having a length $\ell$, a mass per unit length $m$ and a bending rigidity $EI$. The beam is clamed at its right end and has a lumped mass $M_o$ at the left end which supported by a linear spring having a spring constant $k$. By considering free vibration of the beam, write down the boundary conditions and evaluate the frequency determinant.

***Question # 3:***

Express how the displacement spectrum of a seismic record can be obtained. Give mathematical definitions and symbolic variations for displacement, velocity and acceleration spectra for two damping ratios $\xi$. Write down the relationships between these three spectra. Discuss properties of these three spectra for the cases $T \to 0$ and $T \to large$.



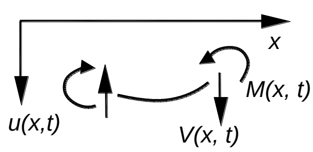




$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t) \qquad \ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g(t) \qquad \omega_D = \omega\sqrt{(1-\xi^2)}$

$$u(t,\xi,\omega) = -\frac{1}{\omega_D}\int_o^t \ddot{u}_g(\tau)\exp[-\xi\omega(t-\tau)]\sin[\omega_D(t-\tau)]\,d\tau$$

$$\dot{u}(t,\xi,\omega) = -\int_o^t \ddot{u}_g(\tau)\exp[-\xi\omega(t-\tau)]\cos[\omega_D(t-\tau)]\,d\tau - \xi\omega\, u(t,\xi,\omega)$$




$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = \left[u_1(t) \quad u_2(t)\right]^T \quad \mathbf{p}(t)^T = \left[P_1(t) \quad P_2(t)\right] \quad \omega_i = 2\,\pi / T_i$

$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \qquad M_i = \boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$

$K_i = \boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \boldsymbol{\phi}_i^T\mathbf{p}(t) \qquad Y_i(t) = \sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}\,/\,M_i \qquad k = \frac{3EI}{h^3} \qquad k = \frac{12EI}{h^3}$

$Y_i(t) = \frac{\sin\omega_i t}{M_i\;\omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right] \qquad L_i = \boldsymbol{\phi}_i^T\mathbf{m}\,\mathbf{1} \quad \Gamma_i = L_i\,/\,M_{\mathrm{i}} \qquad M_i^* = \Gamma_i\,L_i \qquad \mathbf{1} = \left[1 \quad 1\right]^T \qquad V_{bj} = M_j^*\,S_a(T_j)$

$u(x,t) = \sum\boldsymbol{\phi}_i(x)\,Y_i(t) \qquad \ddot{Y}_i(t) + \omega_i^2\;Y_i(t) = 0 \qquad M(x,t) = -EI\,\frac{\partial^2 u}{\partial x^2} \qquad V(x,t) = -EI\,\frac{\partial^3 u}{\partial x^3} \qquad f_{nj} = V_{bj}\,\frac{m_n\,\phi_{nj}}{\sum_k m_k\,\phi_{kj}}$

$\phi(x) = A_1\,\sin ax + A_2\,\cos ax + A_3\,\sinh ax + A_4\,\cosh ax \qquad a^4 = \frac{m\,\omega^2}{EI}$