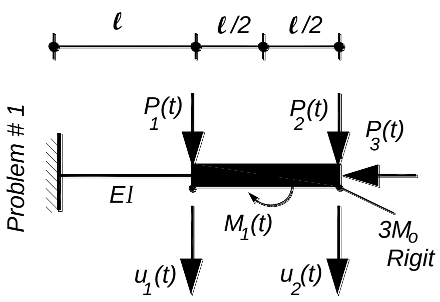
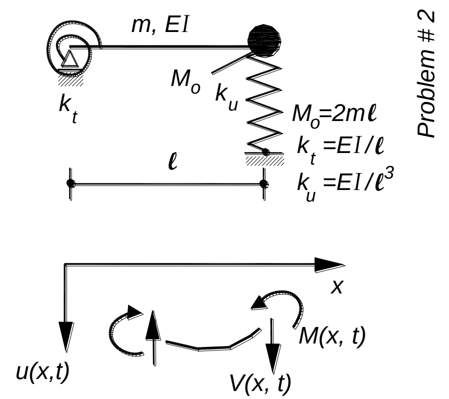
ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / December 04, 2013

Problem # 1:

Consider the system of two degrees-of-freedom shown. (a) Evaluate the flexibility $\mathbf{d}$ matrix by applying forces along the corresponding displacements u_i, Determine the mass matrix $\mathbf{m}$ by transforming inertia forces along the corresponding displacements u_i and the load vector $\mathbf{p}$ by transforming the external forces along the corresponding displacements u_i. Obtain the rigidity matrix $\mathbf{k} = \mathbf{d}^{-1}$. (b) Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, M_o and ℓ. (c) Obtain the corresponding two mode shapes $\boldsymbol{\phi}_i$ and give their graphical representation ($i = 1, 2$). (d) Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m} \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k} \boldsymbol{\phi}_2$. (e) Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m} \boldsymbol{\phi}_i$ and $K_i = \boldsymbol{\phi}_i^T \mathbf{k} \boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2 = K_i / M_i$ ($i = 1, 2$).





Problem # 2:

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The left end of the beam is fixed and has a rotational spring k_t. The beam has a lumped mass of M_o and a lateral spring k_u. Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant in terms of $\beta^4 = m\ell^4\omega^2/(EI)$ by assuming $M_o = 2m\ell$, $k_u = EI/\ell^3$ and $k_t = EI/\ell$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \quad \omega_i = 2\pi/T_i$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \quad M_i = \boldsymbol{\phi}_i^T \mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i = \boldsymbol{\phi}_i^T \mathbf{k}\,\boldsymbol{\phi}_i \quad u(x,t) = \sum \boldsymbol{\phi}_i(x)\,Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \quad M(x,t) = -EI\,\frac{\partial^2 u}{\partial x^2}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \quad a^4 = \frac{m\,\omega^2}{EI} \quad V(x,t) = -EI\,\frac{\partial^3 u}{\partial x^3}$$

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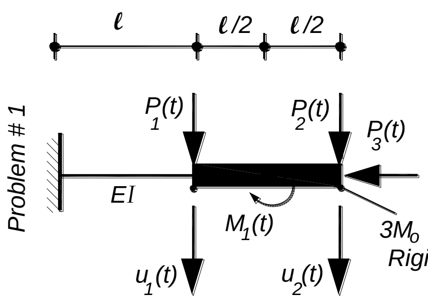

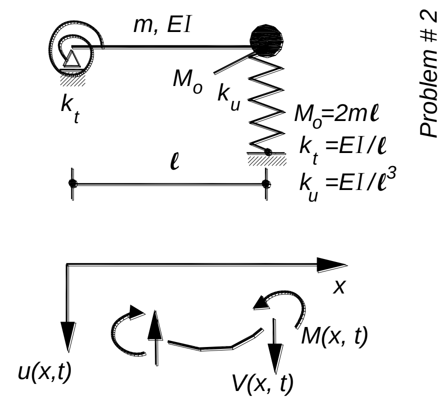

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