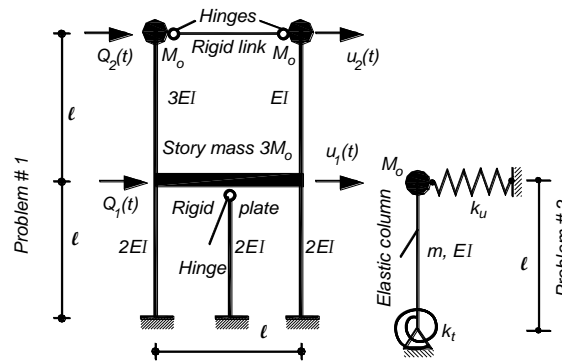


**Problem # 1:**

Consider the system of two degrees-of-freedom shown where the first story is rigid plate having a mass of  $3M_o$  and the second story consists of two cantilever columns connected to each other by a rigid link. (a). Evaluate the flexibility  $\mathbf{d}$  matrix, the mass matrix  $\mathbf{m}$  and the rigidity matrix  $\mathbf{k} = \mathbf{d}^{-1}$  and the load vector  $\mathbf{p}$ . (b). Determine the circular frequencies and the periods of the free vibration  $\omega_i$  and  $T_i$  in terms of  $EI$ ,  $m$  and  $\ell$ . (c). Obtain the corresponding two mode shapes  $\phi_i$  and give their graphical representation ( $i=1,2$ ). (d). Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix  $\phi_1^T \mathbf{m} \phi_2$ , and  $\phi_1^T \mathbf{k} \phi_2$ . e. Evaluate the generalized masses and stiffness  $M_i = \phi_i^T \mathbf{m} \phi_i$  and  $K_i = \phi_i^T \mathbf{k} \phi_i$  and assess the relationship  $\omega_i^2 = K_i / M_i$  ( $i=1,2$ ).

**Problem # 2:**

Consider the elastic beam shown where  $m$  is the mass per unit length and  $EI$  is the bending rigidity of the cross section. The left end of the beam is fixed and has a rotational spring  $k_t$ . The right end of the beam is free and has a concentrated mass  $M_o$  and a lateral spring  $k_u$ . Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant in terms of  $\beta^4 = m \ell^4 \omega^2 / (EI)$  by assuming  $M_o = 2m \ell$ ,  $k_u = 2EI / \ell^3$  and  $k_t = 3EI / \ell$ .



$$\mathbf{m} \ddot{\mathbf{u}}(t) + \mathbf{k} \mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = [u_1(t) \quad u_2(t)]^T \quad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)] \quad \omega_i = 2 \pi / T_i$$

$$(\mathbf{k} - \omega_i^2 \mathbf{m}) \phi_i = 0 \quad (\mathbf{I} - \omega_i^2 \mathbf{d} \mathbf{m}) \phi_i = 0 \quad |\mathbf{k} - \omega_i^2 \mathbf{m}| = 0 \quad |\mathbf{I} - \omega_i^2 \mathbf{d} \mathbf{m}| = 0 \quad M_i = \phi_i^T \mathbf{m} \phi_i$$

$$K_i = \phi_i^T \mathbf{k} \phi_i \quad M_i \ddot{Y}_i(t) + K_i Y_i(t) = \phi_i^T \mathbf{p}(t) \quad Y_i(t) = \sum_{i=1}^2 \phi_i^T \mathbf{m} \mathbf{v} / M_i \quad k = \frac{3EI}{h^3} \quad k = \frac{12EI}{h^3}$$

$$Y_i(t) = \frac{\sin \omega_i t}{M_i \omega_i} \left[ \phi_i^T \int_0^{t_0} \mathbf{p}(\tau) d\tau \right] \quad L_i = \phi_i^T \mathbf{m} \mathbf{1} \quad \Gamma_i = L_i / M_i \quad M_i^* = \Gamma_i L_i \quad \mathbf{1}^T = [1 \quad 1]$$

$$u(x,t) = \sum \phi_i(x) Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2 Y_i(t) = 0 \quad M(x,t) = -EI \frac{\partial^2 u}{\partial x^2} \quad V(x,t) = -EI \frac{\partial^3 u}{\partial x^3}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \quad a^4 = \frac{m \omega^2}{EI}$$

Prof.Dr. Hasan Bodurođlu (<http://www.akademi.itu.edu.tr/bodurogl/>)

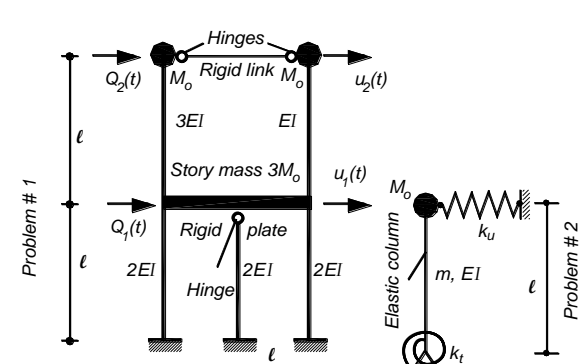
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