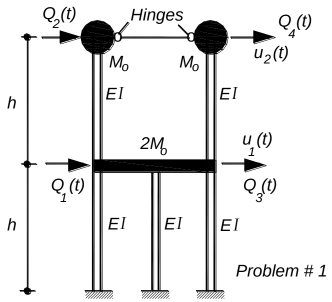
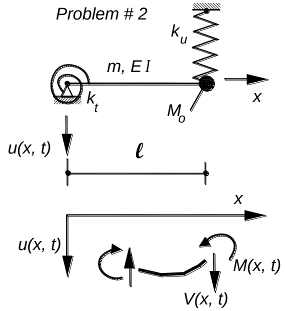
ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / December 13, 2011
H.Boduroğlu / Z.Celep

Problem # 1:
Consider the system of two degrees-of-freedom shown where the first story is rigid plate having a mass of $2M_o$ and the second story consists of two cantilever columns connected to each other by an inextensible rod with negligible mass. a. Evaluate the flexibility $\mathbf{d}$ matrix, the mass matrix $\mathbf{m}$ and the rigidity matrix $\mathbf{k} = \mathbf{d}^{-1}$ and the load vector $\mathbf{p}$. b. Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, M_o and h. c. Obtain the corresponding two mode shapes ϕ_i and give their graphical representation ($i = 1,2$). d. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_2$. e. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T\,\mathbf{m}\,\boldsymbol{\phi}_i$ and $K_i = \boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2 = K_i / M_i$ ($i = 1,2$).





Problem # 2:
Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The left end of the beam is fixed and has a rotational spring k_t. The right end of the beam has a lumped mass of M_o and a lateral spring k_u. Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant in terms of $\beta^4 = m\ell^4\omega^2/(EI)$ by assuming $M_o = m\ell$, $k_u = EI/\ell^3$ and $k_t = EI/\ell$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = \left[u_1(t) \quad u_2(t)\right]^T \quad \mathbf{p}(t)^T = \left[P_1(t) \quad P_2(t)\right] \quad \omega_i = 2\pi/T_i$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \quad M_i = \boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i = \boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i \quad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \boldsymbol{\phi}_i^T\mathbf{p}(t) \quad Y_i(t) = \sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}/M_i \quad k = \frac{3EI}{h^3} \quad k = \frac{12EI}{h^3}$$

$$Y_i(t) = \frac{\sin\omega_i t}{M_i\,\omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right] \quad L_i = \phi_i^T\,\mathrm{m}\,1 \quad \Gamma_i = L_i\,/\,M_\mathrm{i} \quad M_i^* = \Gamma_i\,L_i$$

$$1^\mathrm{T} = [1 \quad 1] \quad u(x,t) = \sum\boldsymbol{\phi}_i(x)\,Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \quad M(x,t) = -EI\,\frac{\partial^2 u}{\partial x^2}$$

$$\phi(x) = A_1\sin ax + A_2\cos ax + A_3\sinh ax + A_4\cosh ax \quad a^4 = \frac{m\,\omega^2}{EI} \quad V(x,t) = -EI\,\frac{\partial^3 u}{\partial x^3}$$

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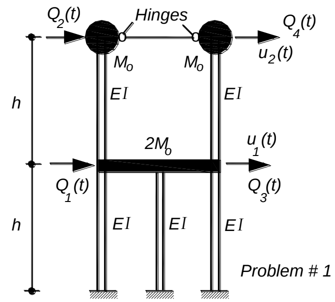

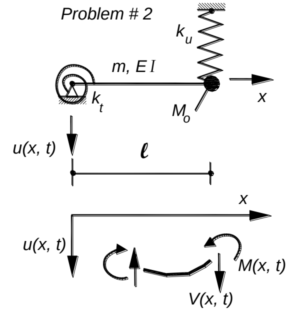

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