ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / December 15, 2010

H.Boduroğlu / Z. Celep

Problem # 1

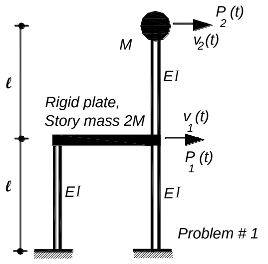
Consider the system of two degrees-of-freedom shown:

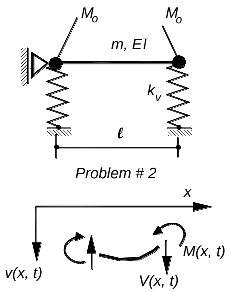
a. Evaluate the flexibility **d** matrix, the mass matrix **m** and the rigidity matrix $\mathbf{k} = \mathbf{d}^{-1}$ and the load vector **p**.

b. Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI , m and ℓ . Obtain the corresponding two mode shapes $\boldsymbol{\phi}_i$ and give their graphical representation (*i =1, 2*),

c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$.

d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess the relationship $\omega_i^{\,2} = K_i / M_i$. (*i =1, 2*),





Problem # 2:

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has two lumped masses of M_o at the two ends. Write down the boundary conditions for the free vibration of the beam. Obtain all parameters in terms of $\beta^4 = \dfrac{m\,\ell^4\,\omega^2}{EI}$ by assuming $M_o = m\,\ell$ and $k_v = E\,I/\ell^3$.

$$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = \begin{bmatrix} v_1(t) & v_2(t) \end{bmatrix}^T \qquad \mathbf{p}(t)^T = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \qquad (\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0$$

$$(\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \qquad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \qquad \omega_i = 2\,\pi / T_i \qquad M_i = \boldsymbol{\phi}_i^T \mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i = \boldsymbol{\phi}_i^T \mathbf{k}\,\boldsymbol{\phi}_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \boldsymbol{\phi}_i^T \mathbf{p}(t) \qquad Y_i(t) = \sum_{i=1}^{2} \boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v} / M_i \qquad k = \frac{3EI}{h^3} \qquad k = \frac{12EI}{h^3}$$

$$Y_i(t) = \frac{\sin\omega_i t}{M_i\,\omega_i}\left[\boldsymbol{\phi}_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad L_i = \phi_i^T\,\mathrm{m}\,1 \qquad \Gamma_i = L_i \,/\, M_\mathrm{i} \qquad M_i^* = \Gamma_i\,L_i$$

$$1^\mathrm{T} = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad \mathbf{v}(x,t) = \sum \boldsymbol{\phi}_i(x)\,Y_i(t) \qquad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \qquad M(x,t) = -EI\,\frac{\partial^2 v}{\partial x^2}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad a^4 = \frac{m\,\omega^2}{EI} \qquad V(x,t) = -EI\,\frac{\partial^3 v}{\partial x^3}$$

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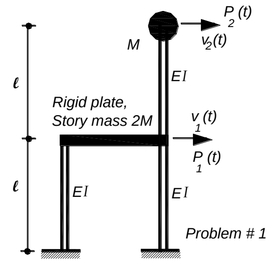

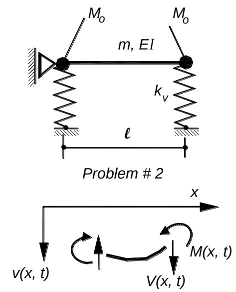

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