ADVANCED DYNAMICS OF STRUCTURES / Midterm Exam / December 16, 2009

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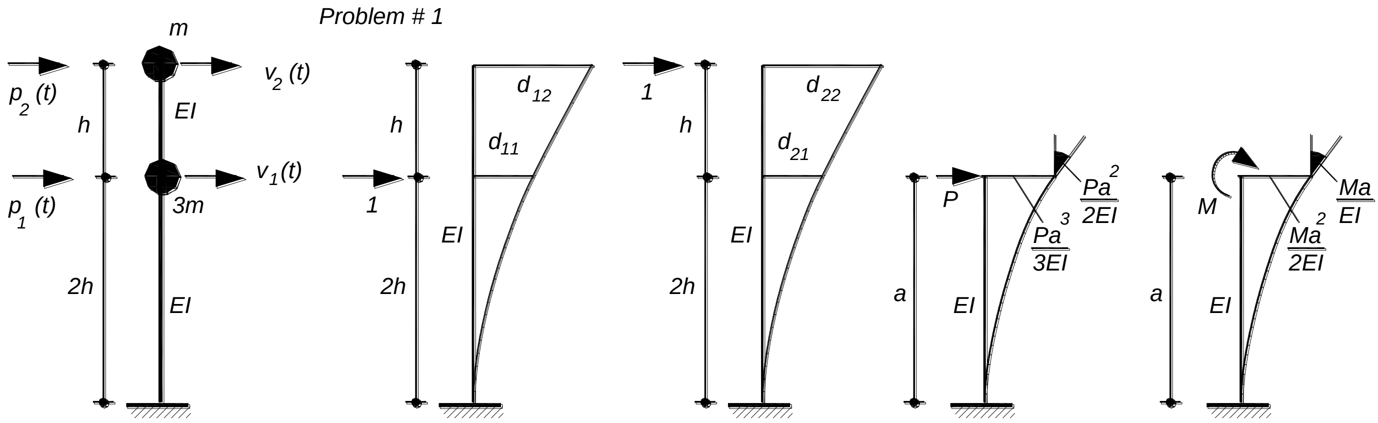
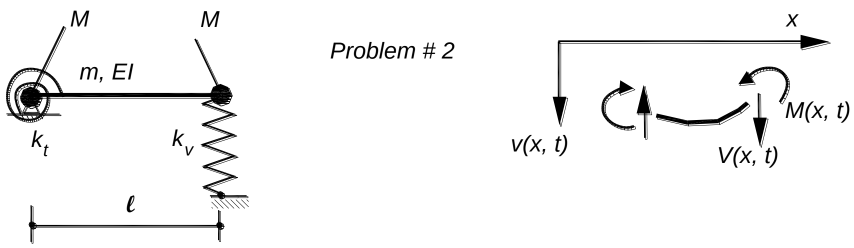
Problem # 1

Consider the system of two degrees-of-freedom shown:

a. Evaluate the flexibility **d** matrix, the mass matrix **m** and the rigidity matrix **k = d**$^{-1}$ and the load vector **p**.
b. Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, m and h. Obtain the corresponding two mode shapes $\boldsymbol{\phi}_i$ and give their graphical representation ($i = 1, 2$),
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$.
d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess the relationship $\omega_i^{\,2} = K_i / M_i$. ($i = 1, 2$),





Problem # 2:

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has two lumped masses of M at the two ends. Write down the boundary conditions for the free vibration of the beam. Obtain all parameters in terms of $\beta^4 = \dfrac{m\,\ell^4\,\omega^2}{EI}$ by assuming $M = m\,\ell$, $k_v = EI/\ell^3$ and $k_t = EI/\ell$.

$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t)$ $\quad$ $\mathbf{v}(t) = [v_1(t) \quad v_2(t)]^T$ $\quad$ $\mathbf{p}(t)^T = [P_1(t) \quad P_2(t)]$ $\quad$ $(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0$ $\quad$ $(\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0$

$\left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0$ $\quad$ $\left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0$ $\quad$ $\omega_i = 2\,\pi / T_i$ $\quad$ $M_i = \phi_i^T \mathbf{m}\,\phi_i$ $\quad$ $K_i = \phi_i^T \mathbf{k}\,\phi_i$ $\quad$ $M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\,\mathbf{p}(t)$

$v(t) = \sum_{i=1}^{2} Y_i(t)\,\phi_i$ $\quad$ $Y_i(t) = \sum_{i=1}^{2} \phi_i^T\,\mathbf{m}\,\mathbf{v} / M_i$ $\quad$ $Y_i(t) = \dfrac{\sin \omega_i t}{M_i\;\omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right]$ $\quad$ $k = \dfrac{3EI}{h^3}$ $\quad$ $k = \dfrac{12EI}{h^3}$

$L_i = \phi_i^T \mathrm{m}\,1$ $\quad$ $\Gamma_i = L_i \,/\, M_i$ $\quad$ $M_i^* = \Gamma_i\;L_i$ $\quad$ $1^{\mathrm{T}} = [1 \quad 1]$ $\quad$ $v(x,t) = \sum \phi_i(x)\,Y_i(t)$ $\quad$ $\ddot{Y}_i(t) + \omega_i^2\;Y_i(t) = 0$

$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax$ $\quad$ $a^4 = \dfrac{m\,\omega^2}{EI}$ $\quad$ $V(x,t) = -EI\,\dfrac{\partial^3 v}{\partial x^3}$ $\quad$ $M(x,t) = -EI\,\dfrac{\partial^2 v}{\partial x^2}$