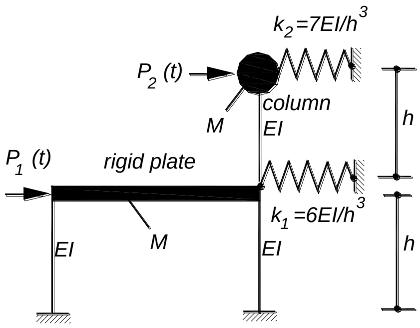
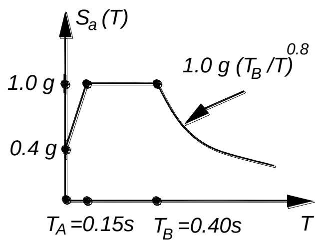
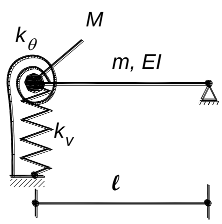
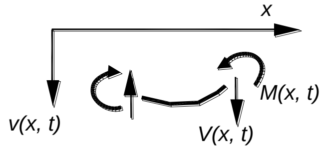
ADVANCED DYNAMICS OF STRUCTURES **Midterm Exam** **December 12, 2007**

1. Consider the system of two-degree-of freedom shown, where the upper part behaves as a cantilever column and the lower part as a shear frame. The column and the shear frame are supported by lateral springs.
   a. Write down equation of motion by including the external forces $P_1(t)$ and $P_2(t)$. Obtain the mass matrix **m** and the stiffness **k** of the system.
   b. Determine the two circular frequencies $\omega_1 < \omega_2$, the two periods of the free vibration and $T_1 > T_2$ and the corresponding mode shapes ϕ_1, and ϕ_2. Give their graphical representations.
   c. Check the orthogonality of the modes with respect to the mass matrix $\phi_1^{\mathbf{T}}\mathbf{m}\,\phi_2$ and the stiffness matrix $\phi_1^{\mathbf{T}}\mathbf{k}\,\phi_2$.
   d. Evaluate the generalized masses and stiffness M_1, M_2 and K_1, K_2 and assess that $\omega_i^2 = K_i / M_i$.
   e. Determine the effective modal masses M_1^*, M_2^* and asses that $M_1^* + M_2^* = 2\,M$ where M is the story mass.
   f. Write down the uncoupled version of the equations of motion as $M_i\,\ddot{Y}_i + K_i\,Y_i = \phi_i^T\,\mathbf{p}$
   g. Assuming that the system is under the impulsive load of P_2 and $I = \int_0^{t_o} P_2(\tau)\,d\tau = P_o\,t_o$ and $P_1(t) = 0$ and assuming that the system starts from the rest, i.e., $\mathbf{v}(t=0) = \dot{\mathbf{v}}(t=0) = 0$ and $t_o << T_2 < T_1$, obtain $Y_i(t)$ and $v_i(t)$.
   h. For a numerical evaluation, assume that $t_o = T_2/10$, $P_o = 2Mg$, $T_1 = 2\sec onds$, calculate T_2 and the maximum amplitudes of v_1 and v_2 by considering the contribution of the first mode only.
   i. By considering the system subjected to earthquake only ($P_1(t=) = 0, P_2(t) = 0$) and the spectral acceleration curve $S_a(T)$ given, evaluate the base shear forces V_{b1} and V_{b2} corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases, the story shear forces and the story displacements. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule. All numerical values will be obtained in terms of Mg.
2. Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The beam has a lumped mass of M at the left end.
   a. Write down the boundary conditions for the free vibration of the beam.
   b. Obtain the frequency determinant.
   c. Obtain the frequency equation and its roots and the numerical values of α_1 and α_2 for the first two frequencies $\omega_i = \alpha_i \sqrt{EI/(m\ell^4)}$ assuming for $M = m\,\ell$ and $k_v = E\,I/\ell^3$ and $k_\theta = E\,I/\ell$
   d. Give the graphical representations of the first two mode shapes.









Important note: Problem 1e – 1i , 2c and 2d are not included in the midterm exam. The solution of the problems (1a - 1h and 2a - 2d) will be collected as homework.

$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\,\mathbf{v}(t) = \mathbf{p}(t)$ $\quad \mathbf{v}(t) = [v_1(t) \quad v_2(t)]^T$ $\quad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)]$ $\quad (\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0$ $\quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0$

$\left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0$ $\quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0$ $\quad \omega_i = 2\,\pi / T_i$ $\quad M_i = \phi_i^T \mathbf{m}\,\phi_i$ $\quad K_i = \phi_i^T \mathbf{k}\,\phi_i$ $\quad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\,\mathbf{p}(t)$

$v(t) = \sum_{i=1}^{2} Y_i(t)\,\phi_i$ $\quad Y_i(t) = \sum_{i=1}^{2} \phi_i^T\,\mathbf{m}\,\mathbf{v} / M_i$ $\quad Y_i(t) = \dfrac{\sin\omega_i t}{M_i\,\omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right]$ $\quad k = \dfrac{3EI}{h^3}$ $\quad k = \dfrac{12EI}{h^3}$

$L_i = \phi_i^T\,\mathrm{m}\,1$ $\quad \Gamma_i = L_i \,/\, M_\mathrm{i}$ $\quad M_i^* = \Gamma_i\,L_i$ $\quad 1^\mathrm{T} = [1 \quad 1]$ $\quad v(x,t) = \sum \phi_i(x)\,Y_i(t)$ $\quad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0$

$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax$ $\quad a^4 = \dfrac{m\,\omega^2}{EI}$ $\quad V(x,t) = -EI\,\dfrac{\partial^3 v}{\partial x^3}$ $\quad M(x,t) = -EI\,\dfrac{\partial^2 v}{\partial x^2}$