Consider the system of two-degree-of freedom shown, where the upper part behaves as a cantilever column and the lower part as a shear frame.

1. Write down equation of motion by including the external forces p_1 and p_2 .Obtain the mass matrix **m** and the stiffness **k** of the system.

2. Determine the two circular frequencies $\omega_1 < \omega_2$, the two periods of the free vibration and $T_1 > T_2$ and the corresponding mode shapes ϕ_1, and ϕ_2. Give their graphical representations.

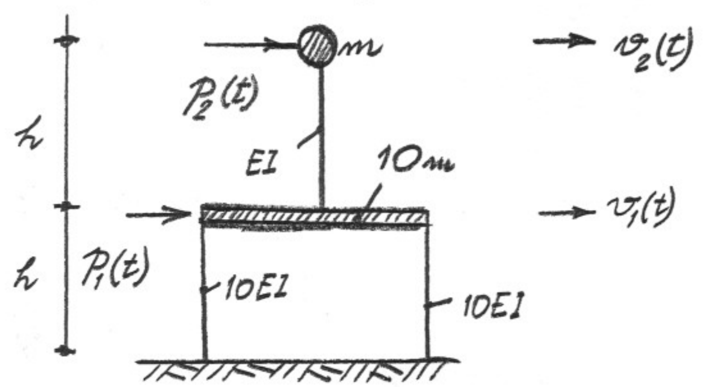
3. Check the orthogonality of the modes with respect to the mass matrix $\phi_1^T \mathbf{m}\, \phi_2$ and the stiffness matrix $\phi_1^T \mathbf{k}\, \phi_2$.

4. Evaluate the generalized masses, stiffness and forces M_1, M_2, K_1, K_2 and P_1, P_2 and assess that $\omega_i^{\,2} = K_i / M_i$.

5. Write down the uncoupled version of the equations of motion as $M_i\, \ddot{Y}_i + K_i\, Y_i = P_i = \phi_i^T\ p_i$

6. Assuming that the system is under the impulsive load of p_1 and $I = \int_0^{t_o} p_1(\tau)\, d\tau = P_o\ t_o$ and $p_2(t) = 0$ and assuming that the system starts from the rest, i.e., $\mathbf{v}(t=0) = \dot{\mathbf{v}}(t=0) = 0$ and $t_o << T_2 < T_1$, obtain $Y_i(t)$ and $v_i(t)$.

7. For a numerical evaluation, assume that $t_o = T_2 / 10$, $P_o = 2mg$, $T_1 = 2$ *seconds*, calculate T_2 and maximum amplitudes of v_1 and v_2 by considering the contribution of the first mode only.



$$\mathbf{m}\,\ddot{\mathbf{v}}(t) + \mathbf{k}\ \mathbf{v}(t) = \mathbf{p}(t) \qquad \mathbf{v}(t) = \begin{bmatrix} v_1(t) & v_2(t) \end{bmatrix}^T \qquad \mathbf{p}(t)^T = \begin{bmatrix} p_1(t) & p_2(t) \end{bmatrix} \qquad (\mathbf{k} - \omega_i^2\ \mathbf{m})\, \phi_i = 0$$

$$(\mathbf{I} - \omega_i^2\ \mathbf{d}\,\mathbf{m})\, \phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\ \mathbf{m}\right| = 0 \qquad \left|\mathbf{I} - \omega_i^2\ \mathbf{d}\,\mathbf{m}\right| = 0 \qquad \omega_i = 2\,\pi / T_i \qquad M_i = \phi_i^T \mathbf{m}\, \phi_i \qquad K_i = \phi_i^T \mathbf{k}\, \phi_i$$

$$M_i\ \ddot{Y}_i(t) + K_i\ Y_i(t) = \phi_i^T \mathbf{p}(t) \qquad \mathbf{v}(t) = \sum_{i=1}^{2} Y_i(t)\, \phi_i \qquad Y_i(t) = \sum_{i=1}^{2} \phi_i^{\,T}\ \mathbf{m}\ \mathbf{v} / M_i$$

$$Y_i(t) = \frac{\sin \omega_i t}{M_i\ \omega_i}\left[\phi_i^T \int_o^{t_o}\ \mathbf{p}(\tau)\ d\tau\right] \qquad k = \frac{3EI}{h^3} \qquad k = \frac{12EI}{h^3}$$

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