ADVANCED DYNAMICS OF STRUCTURES December, 2012 / *HBoduroğlu/ZCelep*

Problem # 1

Consider the system of two degrees-of-freedom shown:

a. Write down the equations of motion of the system by including the external loads.

b. Determine the two circular frequencies and the periods of the free vibration ω_i and T_i and the corresponding mode shapes $\boldsymbol{\phi}_i$. Give their graphical representation ($i = 1, 2$),

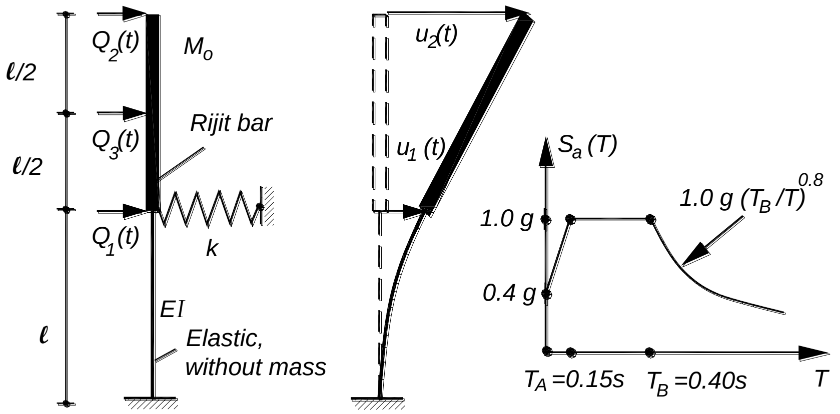
c. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m}\, \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k}\, \boldsymbol{\phi}_2$,

d. Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m}\, \boldsymbol{\phi}_i$, and $K_i = \boldsymbol{\phi}_i^T \mathbf{k}\, \boldsymbol{\phi}_i$, and assess $\omega_i^{\,2} = K_i / M_i$ for $i = 1, 2$,

e. The height of the column $\ell = 3meter$ and their cross section of $b / h = 0.25m \times 0.50m$, the first period of the system is $T_1 = 0.20s$ and $E = 30GPa$. Find the numerical values the parameter M_o and the second period T_2 of the system.

f. Determine the effective modal masses M_1^* and M_2^* and assess that $M_1^* + M_2^* = M_o$

g. Evaluate the base shear forces V_{b1} and V_{b2} and the overturning moments M_{b1} and M_{b2} corresponding to the two mode shapes. Obtain the base shear force V_b and the overturning moment M_b by using the SRSS combination rule.



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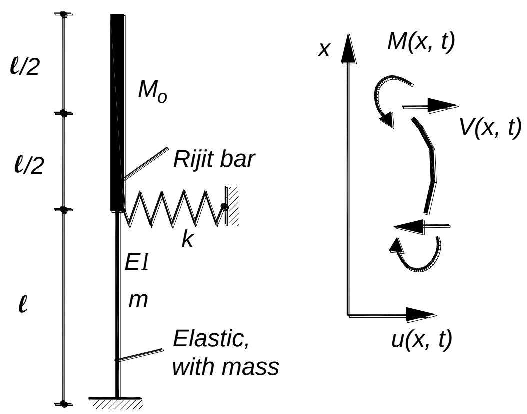

Problem # 2

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Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant by assuming $M_o = m\,a$, $k = E\,I / \ell^3$ in terms of β where $\beta^4 = \dfrac{m\,\ell^4\,\omega^2}{EI}$

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \qquad \mathbf{u}(t) = \left[u_1(t) \quad u_2(t)\right]^T \qquad \mathbf{p}(t)^T = \left[P_1(t) \quad P_2(t)\right] \qquad (\mathbf{k} - \omega_i^2\,\mathbf{m})\,\phi_i = 0 \qquad \omega_i = 2\,\pi / T_i$$

$$(\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\phi_i = 0 \qquad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \qquad K_i = \phi_i^T\,\mathbf{k}\,\phi_i \qquad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \phi_i^T\,\mathbf{p}(t) \quad \mathbf{u}(x,t) = \sum \phi_i(x)\,Y_i(t)$$

$$\ddot{Y}_i(t) + \omega_i^2\;Y_i(t) = 0 \qquad Y_i(t) = \frac{\sin\omega_i t}{M_i\;\omega_i}\left[\phi_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \qquad Y_i(t) = \phi_i^T\,\mathbf{m}\,\mathbf{v} / M_i$$

$$M_i = \phi_i^T\,\mathbf{m}\,\phi_i \qquad M(x,t) = -EI\,\frac{\partial^2 u}{\partial x^2} \qquad V(x,t) = -EI\,\frac{\partial^3 u}{\partial x^3} \qquad a^4 = \frac{m\,\omega^2}{EI}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \qquad L_i = \phi_i^T\,\mathbf{m}\,\mathbf{1} \qquad \Gamma_i = L_i \;/\; M_i \qquad M_i^* = \Gamma_i\;L_i$$

$$I_\theta = \frac{M}{12}(a^2 + b^2) \qquad V_{b\,j} = M_j^*\;A_{j\,max} \qquad f_{ij} = V_{b\,j}\,\frac{m_i\;\phi_{ij}}{\sum_{l=1}^{n} m_l\;\phi_{lj}} \qquad M_j^* = \frac{\left(\sum_{i=1}^{n} m_i\;\phi_{ij}\right)^2}{\sum_{i=1}^{n} m_i\;\phi_{ij}^2}$$