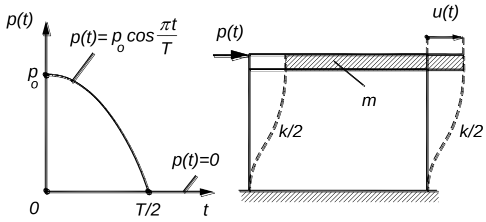
ADVANCED DYNAMICS OF STRUCTURES / April 2014

Problem # 1:

A single degree of freedom system of the mass m, the stiffness k is subjected to the external load $p(t)$, having a variation as shown. Assuming the system starts from the rest position, i.e., $u(t=0)=0$ and $\dot{u}(t=0)=0$, find the displacement function $u(0 \le t \le T/2)$ by using the initial conditions and $u(t \ge T/2)$ by using the continuity of the displacement and the velocity, where T is the free vibration period of the system.

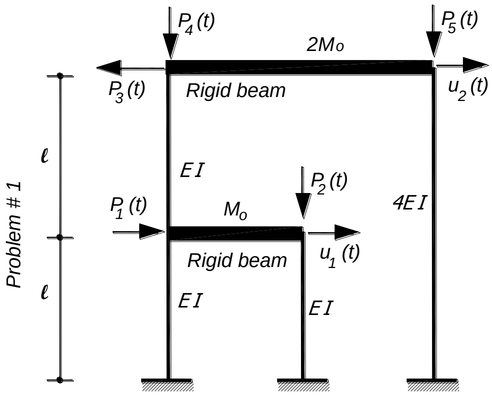


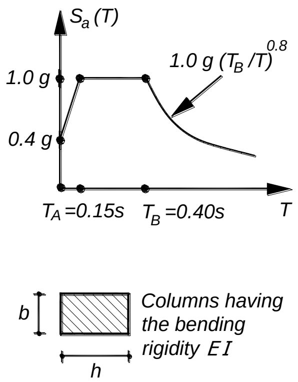
Problem # 2:

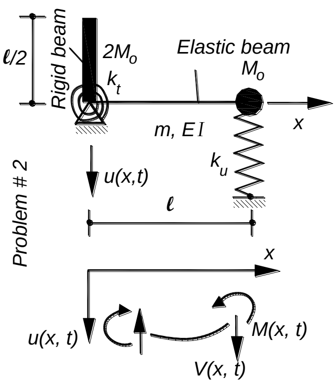
a. Consider the system of two degree-of-freedom shown where the first and the second stories are rigid plates having a mass of $3M_o$ and M_o, respectively. (a) Write down equations of motion by considering the free body diagram of the two story masses separately. (b) Evaluate the mass matrix $\mathbf{m}$, and the rigidity matrix $\mathbf{k}$ and the load vector $\mathbf{p}$. (c) Determine the circular frequencies ω_i and the periods T_i of the free vibration in terms of EI, M_o and ℓ. (d) Obtain the corresponding two mode shapes ϕ_i and give their graphical representation ($i=1,2$). (e) Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_2$. (f). Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T\,\mathbf{m}\,\boldsymbol{\phi}_i$ and $K_i = \boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2 = K_i / M_i$ ($i=1,2$). Determine the effective modal masses M_1^* and M_2^*, and assess $M_1^* + M_2^* = 3M_o$

b. The heights of the stories are $\ell = 3meter$, the columns have cross section of $b/h = 0.25m/0.50m$, the weight $M_o g = 400kN$ and $E = 30GPa$. Find the first and second periods T_1 and T_2 of the system.

c. Evaluate the base shear forces V_{b1} and V_{b2} corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases and the story shear forces by using the acceleration spectrum given. Obtain the story forces f_{21} and f_{22} due to the base shear forces V_{b1} and V_{b2} and the bending moments at the columns of the second story. Obtain the shear force at the second story by using the SRSS combination rule.







Problem # 3:

Consider an elastic beam having a cross sectional bending rigidity having EI, a mass per unit length m and a length ℓ. The left end of the elastic beam is simply supported having a rotational spring with a spring constant k_t and a vertical rigid beam with a mass $2M_o$; its right end has a mass M_o and it is connected to a lateral spring having a spring constant k_u. Write down the boundary conditions for the free vibration of the system. Obtain the frequency determinant in terms of $\beta^4 = (a\ell)^4 = m\,\ell^4\omega^2/(EI)$ by assuming $M_o = 2m\ell$, $k_u = 2EI/\ell^3$. and $k_t = 2EI/\ell$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = [u_1(t) \quad u_2(t)]^T \quad \mathbf{p}(t)^T = [P_1(t) \quad P_2(t)] \quad \omega_i = 2\,\pi/T_i$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \quad M_i = \boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i = \boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i \quad M_i\,\ddot{Y}_i(t) + K_i\,Y_i(t) = \boldsymbol{\phi}_i^T\mathbf{p}(t) \quad Y_i(t) = \sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}/M_i \quad k = \frac{3EI}{h^3} \quad k = \frac{12EI}{h^3} \quad k = \frac{12EI}{h^3}$$

$$Y_i(t) = \frac{\sin\omega_i t}{M_i\;\omega_i}\left[\boldsymbol{\phi}_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \quad L_i = \boldsymbol{\phi}_i^T\mathbf{m}\,\mathbf{1} \quad \Gamma_i = L_i\,/\,M_i \quad M_i^* = \Gamma_i\,L_i \quad \mathbf{1} = [1 \quad 1]^T \quad V_{bj} = M_j^*\,S_a(T_j)$$

$$u(x,t) = \sum \boldsymbol{\phi}_i(x)\,Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2\,Y_i(t) = 0 \quad M(x,t) = -EI\frac{\partial^2 u}{\partial x^2} \quad V(x,t) = -EI\frac{\partial^3 u}{\partial x^3} \quad f_{nj} = V_{bj}\frac{m_n\,\phi_{nj}}{\sum_k m_k\,\phi_{kj}}$$

$$\phi(x) = A_1\sin ax + A_2\cos ax + A_3\sinh ax + A_4\cosh ax \quad a^4 = \frac{m\,\omega^2}{EI}$$