PROJECT # 1

The incompressible Navier-Stokes equations that govern the incompressible viscous fluid flow in the Cartesian coordinate system can be written in dimensionless form as follows: the momentum equations

$$Re\frac{\partial u}{\partial t} + Re\frac{\partial (uu)}{\partial x} + Re\frac{\partial (uv)}{\partial y} + \frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$Re\frac{\partial v}{\partial t} + Re\frac{\partial (uv)}{\partial x} + Re\frac{\partial (vv)}{\partial y} + \frac{\partial p}{\partial y} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \tag{2}$$

the continuity equation

$$-\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{3}$$

In these equations (u, v) represents the velocity vector components, p is the pressure and Re is the dimensionless Reynolds number. The primitive variables can be arranged as shown in Figure 1. The finite-difference approximations to the momentum equations (1) and (2) can be written: the momentum equations along the x-axis

$$Re\frac{u^{n+1}_{i+1/2,k} - u_{i+1/2,k}}{\Delta t} + Re\frac{(uu)_{i+1,k} - (uu)_{i,k}}{\Delta x} + Re\frac{(uv)_{i+1/2,k+1/2} - (uv)_{i+1/2,k-1/2}}{\Delta y}$$

$$+\frac{p_{i+1,k} - p_{i,k}}{\Delta x} = \frac{u_{i+3/2,k} - 2u_{i+1/2,k} + u_{i-1/2,k}}{\Delta x^2} + \frac{u_{i+1/2,k+1} - 2u_{i+1/2,k} + u_{i+1/2,k-1}}{\Delta y^2} \tag{4}$$

the momentum equations along the y-axis

$$Re\frac{v^{n+1}_{i,k+1/2} - v_{i,k+1/2}}{\Delta t} + Re\frac{(uv)_{i+1/2,k+1/2} - (uv)_{i-1/2,k+1/2}}{\Delta x} + Re\frac{(vv)_{i,k+1} - (vv)_{i,k}}{\Delta y}$$

$$+\frac{p_{i,k+1} - p_{i,k}}{\Delta y} = \frac{v_{i+1,k+1/2} - 2v_{i,k+1/2} + v_{i-1,k+1/2}}{\Delta x^2} + \frac{v_{i,k+3/2} - 2v_{i,k+1/2} + v_{i,k-1/2}}{\Delta y^2} \tag{5}$$

The similar approximations to the continuity equation (3)

$$-\frac{u_{i+1/2,k} - u_{i-1/2,k}}{\Delta x} - \frac{v_{i,k+1/2} - v_{i,k-1/2}}{\Delta y} = 0 \tag{6}$$

The no-slip boundary conditions can be applied using ghost cells within the solid domain as shown in Figure 2. The application of $u_b = 0$ requires that $u_{i+1/2,k} = 0$. In a similar manner, the application of $v_b = 0$ requires that $v_{i+1,k+1/2} = -v_{i,k+1/2}$.

Please use the above described MAC scheme [2] to solve the Stokes flow within the unit square $[-0.5, 0.5] \times [-0.5, 0.5]$. In the second part of the project, please solve the lid-driven cavity flow in a square enclosure $[-0.5, 0.5] \times [-0.5, 0.5]$ at $Re = 1000$ (This part is optional). The boundary conditions are

$$u(x, +0.5) = 1 \quad , \quad v(x, +0.5) = 0 \tag{7}$$
$$u(-0.5, y) = 0 \quad , \quad v(-0.5, y) = 0 \tag{8}$$
$$u(+0.5, y) = 0 \quad , \quad v(+0.5, y) = 0 \tag{9}$$
$$u(x, -0.5) = 0 \quad , \quad v(x, -0.5) = 0 \tag{10}$$

Once you compute the numerical solution please compare your u-velocity component along the centerline of the cavity ($x = 0$) with the results of Botella and Peyret [1]. In addition to these validation graphs, please provide the contour plots of primitive variables for all cases. During the coding, please try to use the local numbering given in Figure 3.

[0]ISTANBUL TECHNICAL UNIVERSITY, UUM601E, 31 October 2012

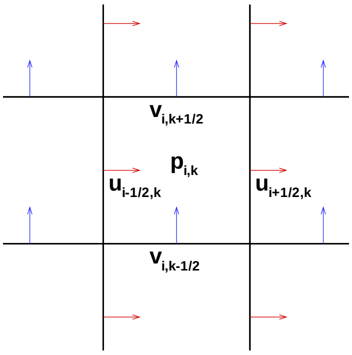

Figure 1: The arrangement of primitive variables.

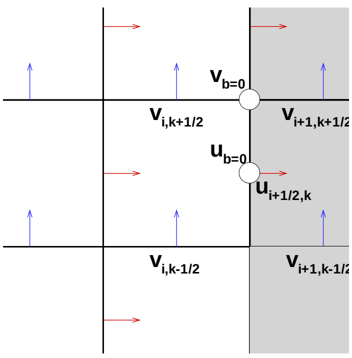

Figure 2: The application of no-slip boundary condition.

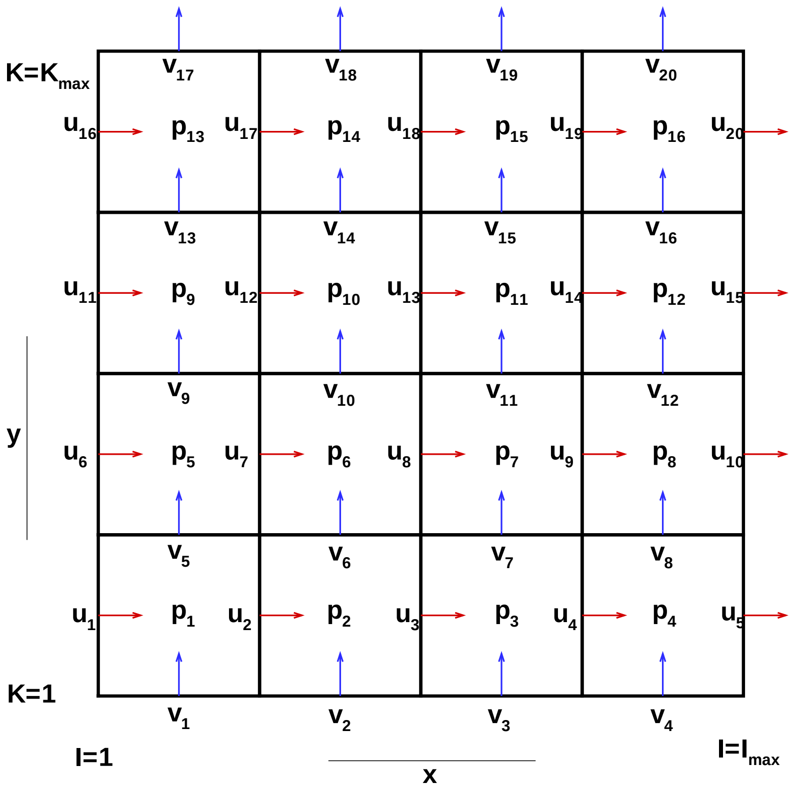

Figure 3: The local numbering of primitive variales

References

[1] O. Botella and R. Peyret, Benchmark spectral results on the lid-driven cavity flow. *Computers & Fluids* **27**:421-433, (1998).

[2] F. H. Harlow and J. E. Welch, Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface. *J. Comput. Phys.* **8**:21822189, (1965).