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Application of an Evolutionary Algorithm for VaR Calculations

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Abstract: - The Value-at-Risk (VaR) approach has been extensively used for measuring and controlling of market risks in financial institutions during the last decade. The risk control and management systems required in the new banking industry are based on the Banks for International Settlements' *(BIS)* suggestions. Financial asset returns are traditionally modeled as being distributed according to the normal or lognormal distributions. However the VaR estimations, calculated in this way, usually involve a systematic error because the density of the returns' occurrences is not distributed normally. The leptokurtic distribution of financial asset returns can be defined more realistically with a t-distribution. The aim of this study is to estimate the parameters of t-distribution through Maximum Likelihood Estimation *(MLE)* using an Evolutionary Algorithm *(EA)* approach. Experimental results show that successful VaR calculations at high confidence levels can be done using the t-distribution with the parameter setting found by the EA.

Key-Words: VaR, Evolutionary Algorithm, Maximum Likelihood Estimation, Student's t-Distribution

1 Introduction

Risk managers and regulators need measures for risk. One of the most popular and well known measures of risk is Value-at-Risk (*VaR*). VaR gives an upper bound for the money to be lost for a given probability, usually taken as 95%, 90%, 99% or 99.9%. The traditional way is to assume the distribution of the returns to be normal or lognormal. However, in practice this assumption seldom holds, because the tails are thicker than in a normal distribution. One possible alternative is to use the Student's t-distribution, which has fat tails. The degrees of freedom of the t-distribution do not need to be integer. The Student's t-distribution offers a very tractable distribution that accommodates fat-tails. As its degrees of freedom increase, the t-distribution also converges to the normal, so it can be seen the t-distribution as a generalization of the normal distribution which usually has fatter tails [1].

In this paper degrees of freedom was used as non-integer. A nice property of this class of distributions is that kurtosis and degrees of freedom have a simple relationship. Parameters of the t-distribution can be estimated through a Maximum Likelihood Estimation *(MLE*). In this study, the Log-Likelihood maximization is achieved through an Evolutionary Algorithm *(EA)* approach. Using an EA to solve this optimization for estimating the Student's t-distribution parameters is straightforward.

In this study, three different shares close prices were used from the Istanbul Stock Exchange *(ISE)* in the between period 01.01.1994 and 25.11.2005 data [2].

This paper is organized as follows: Section 2 introduces the VaR measure and provides an overview of the problem. In section 3, the MLE technique is explained. Section 4 gives a brief look into EAs. In section 5, the experimental design is outlined and results of the experiments are provided. Section 6, perform the experimental results and section 7 concludes the paper and provides possible extensions to the current study.

2 Value-at-Risk

Risk under the VaR model is defined as the maximum expected loss at a certain confidence level over a given period of time [3]. The most well-known VaR model was developed by JP Morgan named as riskmetrics [4]. The VaR definition is based on two fundamental elements; holding period (1- or 10-working day) and confidence level (95% or 99%). The Basel Committee suggests that 10-working days and 99% confidence level may be used in VaR computations while JP Morgan suggests that 1-working day and 95% confidence levels could be employed. The following subsections describe the VaR techniques used in this study.

2.1 Parametric VaR with Normal Distribution's Approach

The VaR associated with normally distributed log returns is [5];

$$VaR(h) = P - P^* = P - e^{\mu h + \alpha\sigma\sqrt{h} + \ln P} \quad (1)$$

where P is the current value of portfolio, P^* is the $(1-CL)$ percentile (or critical percentile) of the terminal value of the portfolio after a holding period of h days. α is the standard normal variate associated with chosen confidence level (e.g., so $\alpha = -1.645$ if a confidence level is 95%).

2.2 Parametric VaR with *t*-Distribution's Approach

The VaR associated with the t-distribution method is [5];

$$VaR(h) = P - P^* = P - e^{\mu h + \alpha_{df}\sigma\sqrt{h} + \ln P} \quad (2)$$

where P is the current value of portfolio, P^* is the $(1-CL)$ percentile (or critical percentile) of the terminal value of the portfolio after a holding period of h days. α_{df} is the Student's t variate corresponding to the chosen confidence level, and df is the number of degrees of freedom [5].

2.3 Monte-Carlo Simulation Method

Computation of VaR under Monte-Carlo *(MC)* simulation includes four steps; first, volatilities and correlations among risk factors are computed; second, expected price/rates under the chosen distribution using computed volatilities are produced; third, random expected prices are produced; finally, the value of a portfolio by using the computed prices are calculated.

In this study the MC VaR was computed both with the normal and the t-distribution assumptions for single-stock portfolios.

2.4 Historical VaR Method

This approach can be seen as a simplified MC Simulation method. In this model, historical data are used to produce scenarios. Therefore, the assumption of normality and the computation of volatility and correlations are not required.

In this study the Historical VaR was computed for comparing with the results obtained with the other assumptions.

3 Maximum Likelihood Estimation

If x is a continuous random variable with pdf *(probability density function)* $pdf = f(x;\theta_1,\theta_2,...,\theta_k)$ where θ_k are the unknown parameters for the N independent observations $x_1, x_2,..., x_N$. The Likelihood Function is [6],

$$L(x_1,x_2,..x_n \mid \theta_1,\theta_2,...,\theta_k) = \prod_{i=1}^{N} f(x_i;\theta_k) \quad (3)$$

and the logarithmic likelihood function is;

$$\Lambda = \ln L = \sum_{i=1}^{N} \ln f(x_i;\theta_1,\theta_2,...,\theta_k) \quad (4)$$

MLE of $\theta_1,\theta_2,...,\theta_k$ are obtained by maximizing L or by maximizing Λ, which is much easier to work with than L [6].

In this study, the t-distribution parameters were estimated with MLE. To perform the MLE for the t-distribution, the following steps should be done.

Start with the pdf of the t-distribution given as:

$$f(x;\nu,\mu,\gamma) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\gamma\sqrt{\pi\nu}\left(1+(\frac{x-\mu}{\gamma\sqrt{\nu}})^2\right)^{\frac{(\nu+1)}{2}}} \quad (5)$$

where μ is a location parameter, γ is a scale parameter, and ν is a shape parameter (degrees of freedom). The standard t-distribution assumes $\mu = 0$, $\gamma = 1$, and ν to be an integer. Then the likelihood function is given by

$$L(x_1,x_2,...x_n \mid \nu,\mu,\gamma) = \prod_{i=1}^{N} \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\gamma\sqrt{\pi\nu}\left(1+(\frac{x-\mu}{\gamma\sqrt{\nu}})^2\right)^{\frac{(\nu+1)}{2}}} \quad (6)$$

From this, the log-likelihood function is obtained as

$$\Lambda = N[\log\Gamma(\frac{\nu+1}{2}) - \log\Gamma(\frac{\nu}{2}) - \frac{1}{2}\log\pi\nu - \log\gamma] - \frac{(\nu+1)}{2}\sum_{i=1}^{N}\log(1+(\frac{r_i-\mu}{\gamma\sqrt{\nu}})^2) \quad (7)$$

Unlike for the normal distribution, no analytical expressions are available for the maximum log-likelihood estimates of ν, μ, γ [7]. Thus use of numerical techniques became necessity. In this study, the experiment was chosen with using an EA for this purpose.

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4 Evolutionary Algorithms

Among the set of population-based search and optimization heuristics, the development of Evolutionary Algorithms *(EA)* [8] has been very important in the last decade. EAs are used successfully in many applications of high complexity.

"Evolutionary Algorithms" [8] is a term that covers a group of heuristic approaches to problem solving using models of natural mechanisms and principles based mainly on Darwin's theory of evolution and the Mendelian principles of classical genetics. EAs work on a population of individuals, each of which represents a solution to the problem and they use an iterative, stochastic search process which is guided based on the goodness or badness of current solutions. EAs go through three main stages in one iteration (generation): selection, recombination and mutation. Selection ensures that individuals with better characteristics are chosen to produce new individuals. Some reproductive mechanisms (cross-over and mutation mostly) are modeled and applied to the selected individuals. At the end of each iteration, some or all of the new individuals replace the old ones. Through these mechanisms, an EA has the power to explore new solutions and exploit those that have already been found. EAs have been increasingly used in many areas in business, engineering and scientific applications. They have been used individually as well as combined with others to solve many problems which are harder to solve using traditional methods. EAs are different than other traditional optimization algorithms mainly in the following ways: Firstly, unlike most traditional methods which start from a single point and work toward better solutions using some transition rules, EAs start searching from a population of points in parallel. This allows an EA to climb many peaks in parallel which reduces the possibility of getting stuck at a local optimum. Furthermore, EAs use fitness information, not derivatives or other time consuming or hard to obtain auxiliary information which makes them easier to implement and apply to various problem domains. These main differences from traditional methods give EAs the robustness expected of good optimization procedures. Further detailed info on EAs can be found in [9].

5 Experimental Design

5.1 Setting up the EA

The fitness function used by the EA to maximize the log-likelihood function is as follows;

$$\Lambda = N[\log\Gamma(\frac{\nu+1}{2}) - \log\Gamma(\frac{\nu}{2}) - \frac{1}{2}\log\pi\nu - \log\gamma] - \frac{\nu+1}{2}\sum_{i=1}^{N}\log(1+(\frac{r_i-\mu}{\gamma\sqrt{\nu}})^2) \quad (8)$$

and the numerical optimization problem to maximize;

$$\max : \Lambda(\nu,\mu,\gamma)$$
$$s.t.$$
$$2.1 \le \nu \le 20.5$$
$$0.0001 \le \mu \le 1.5$$
$$0.001 \le \gamma \le 2.5$$

The assumptions for the ranges for parameters were followed from a similar study [7]. In this study a standard implementation was used for an EA. The general algorithmic flow of the EA is given below;

```
generate initial population;
evaluate initial population;
repeat
        select pairs;
        recombine pairs;
        apply mutation;
        evaluate population;
        do elitism;
until endOfGenerations;
```

The parameters and operators chosen for EA used in this study are given in Table 1.

Table 1. Description of the EA for the maximization of log-likelihood function.

Representation	Floating point
Parent Selection	Tournament selection ts=2
Recombination	Uniform crossover
Crossover Probability	0.8
Mutation	Gauss mutation
Mutation Probability	100%
Survival Selection	Generational
Number of Generations	5000
Population Size	100
Chromosome Size	3
Number of runs	20
Initialization	Random
Elitism	Yes

A chromosome consists of the three parameters of the t-distribution which are ν, μ and γ. For the initial population generation and also during mutation, the lower and upper bounds for the parameters are taken into consideration. When performing the generational elitism, the previous best individual of the population is replaced with the worst individual of the current population. For gauss mutation, different mutation step sizes (standard deviation of the Gaussian distribution) were assumed for each parameter as given below.

$$\nu \Rightarrow N(0,\sigma_\nu) \qquad \sigma_\nu = 1$$
$$\mu \Rightarrow N(0,\sigma_\mu) \qquad \sigma_\mu = 0.0001$$
$$\gamma \Rightarrow N(0,\sigma_\gamma) \qquad \sigma_\gamma = 0.01$$

In this study GNU Scientific Library *(GSL)* was used for generating all random numbers [10].

5.2 Applying the MC VaR

A critical part of a MC simulation is the generation of random variables. Firstly random numbers are generated from the standard form of distributions. In this study, the GSL Random Number Generator Library was used to generate the pseudo random numbers [10].

5.2.1 Normal Random Number Generator

The pseudo normal random numbers (r_n) for each MC simulation are generated as;

$$r_n = \mu + \sigma Z \qquad Z \sim N(0,1)$$

where; μ is the mean of asset returns, σ is standard deviation of the asset returns, and Z is a standard normal random number which is generated by GSL.

5.2.2 t Random Number Generator

The t random number (r_t) for each MC simulations are generated as;

$$r_t = \mu + \gamma Z \qquad Z \sim T(\nu)$$

where; μ is the location parameter, γ is the scale parameter, ν is the non-integer degrees of freedom, $T(\nu)$ is a number from the standard t-distribution (assumes $\mu = 0$, $\gamma = 1$). Z is a standard t-distributed random number variable with respect to ν which is generated by GSL.

6 Experimental Results

μ, γ and ν for the three shares data are estimated using the EA described in the previous section.

The parameters reported in the tables are the best values obtained from the 20 runs of the EA. It is possible to use the best values over all the runs for the VaR calculations since the standard deviations of the mean best fitnesses are very low as can be seen in the table below. The plots show that the numbers of generations for the current settings of the EA are sufficient to provide good convergence. Longer runs are not required.

Table 2. Estimated parameter of t-distribution with EA and standard deviations of mean best fitness.

Share	ν	μ	γ	$\sigma_{fitness}$
DOHOL	3.6190	0.0018	0.0294	**0.1202%**
KCHOL	3.1886	0.0016	0.0240	**0.3035%**
VESTL	3.0649	0.0014	0.0260	**3.3125%**

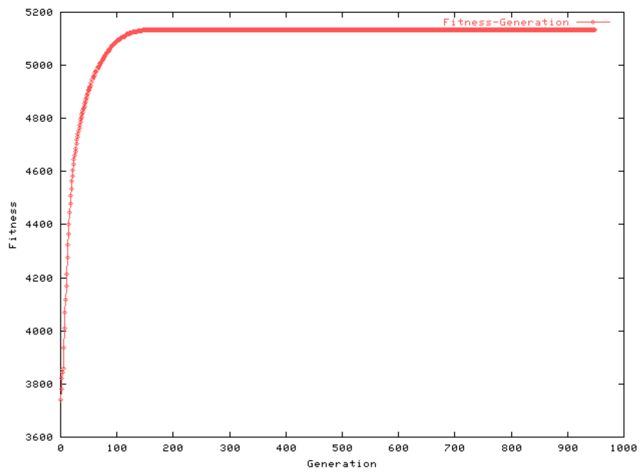

Fig.1 Mean best fitness over the 20 runs on DOHOL share

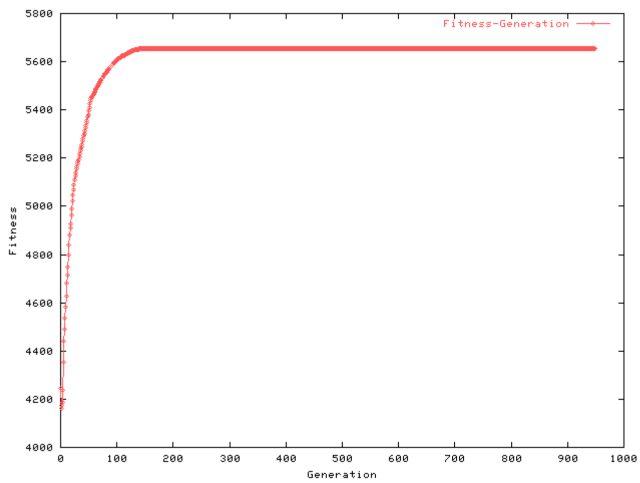

Fig.2 Mean best fitness over the 20 runs on KCHOL

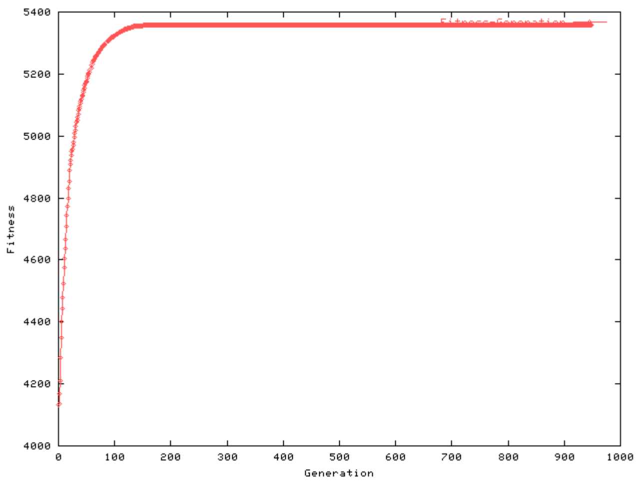

Fig.3 Mean best fitness over the 20 runs on VESTL

The obtained parameters of the t- distribution were applied to the MC simulations and results were obtained for the five different VaR method implementations.

Table.3, Table.4 and Table.5 these results of DOHOL, KCHOL and VSTL shares of ISE index. In all tables, the *Prmt-for-Normal* is computed with the Parametric VaR with normal distribution's approach, *Prmt-for-*t are computed with the Parametric VaR with t-distribution's approach, *RealVaR* is computed with Historical VaR method, *MC-for-Normal* is computed with the Monte-Carlo simulation method for normal distribution and *MC-for-*t is computed with Monte-Carlo simulation method for t-distribution.

Table 3. VaR values for five different methods of DHOL share with different confidence levels.

DOHOL	90%	95%	99%	99.9%
Prmt-for-Normal	5.1976	6.6648	9.3556	12.2801
Prmt -for-t	4.2486	5.940	10.2993	18.9194
RealVaR	**4.4117**	**6.4561**	**10.9243**	**17.9620**
MC-for-Normal	5.1502	6.6285	9.3891	12.2078
MC-for-t	4.3079	5.9808	10.5583	18.5561

In actual financial applications, VaR values for high confidence levels are usually preferred. As can be seen from the results, the VaR values calculated using the t-distribution whose parameters were optimized using an EA are the best in each case. This shows that the assumption of normality on the returns data is not very realistic and gives a systematic error.

Table 4. VaR values for five different methods of KCHOL share with different confidence levels.

KCHOL	90%	95%	99%	99.9%
Prmt-for-Normal	4.3150	5.5500	7.8236	10.3074
Prmt–for-t	3.4677	4.8500	8.4809	15.7318
RealVaR	**3.5826**	**5.2284**	**9.4060**	**14.0956**
MC-for-Normal	4.2858	5.4887	7.7878	10.3874
MC-for-t	3.6830	5.1022	9.3875	15.9566

Table 5. VaR values for five different methods of VESTL share with different confidence levels.

VESTL	90%	95%	99%	99.9%
Prmt-for-Normal	4.8959	6.2916	8.8544	11.6441
Prmt–for-t	3.7751	5.2637	9.1663	16.9212
RealVaR	**3.9604**	**5.7566**	**10.8696**	**17.1514**
MC-for-Normal	4.9186	6.3256	8.8400	11.6701
MC-for-t	3.9380	5.6785	10.5742	17.6365

7 Conclusions and Future Studies

In this study, it demonstrated that the assumption of t-distribution estimated by EA is far better than the assumption of normal distribution in terms of the proximity of the corresponding results to the historical VaR. This is particularly true for higher confidence levels, which are commonly used in the financial industry.

A further extension of this study may be the utilization of parallel algorithms for MC simulations, which prove to be very time consuming. Actually, the parametric VaR figures with the assumption of t-distribution are shown to be sufficiently reliable to be used in practice. Another potential area for future studies is the implementation of different EA techniques for improving the EA performance.

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